THE UNIVERSITY OF MISSOURI STUDIES

CONTENTS

THE MUSICIAN'S ARITHMETIC

DRILL PROBLEMS FOR AN INTRODUCTION TO THE SCIENTIFIC STUDY OF MUSICAL COMPOSITION

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use of numbers. Exercises with figured basses are admitted to be a splendid method of training a musician to be capable of improvising on the organ at a moment's notice. If a musician *approves* of that training, he is likely to conclude that he already possesses all the good which can come out of *numbers*. He needs no more. Why waste his time on further learning? If he *disapproves* of training by figured basses, he is likely to conclude that all *numbers* are damnable. In either case he will overlook the fact that the ratios of vibration rates serve the purpose, quite different from facilitating the visualization of hand movements over suitable organ keys, of facilitating a twofold discussion —the discussion of the "wherefore" both *in the history of the past and in the probability of the future* of music.

Second, a musician may feel "instinctively" (and, more or less, rightly) that an intimate acquaintance with the part of physics called acoustics gives no information essential to him who is interested in the scientific explanation of the *psychological* and *sociological* phenomenon called music. (As to the technique of instrument building, that is a different matter, of course.) The present writer in his younger years specialized in the study of physics and knows what he is talking about. He above said "instinctively" in order to indicate his conviction that such a musician, even though perhaps not knowing what he is talking about, is very nearly right. By acoustics, let us admit, one cannot for music reconstruct the past nor foretell the possibilities and the limitations of the future. Now, a musician might conclude that numbers, since they are the symbols of vibration rates, symbols of physics, can be of little use, except, maybe, for the instrument builder using principles of acoustics. But the numbers of vibration rates are of significance for musical theory not for physical, but for physiological-chemical reasons. Our *nervous functions* depend on those ratios when we hear music. The musician who abhors numbers, therefore abhors the way leading toward an understanding of musicianship, an understanding of the psychology of music.

Third, *most* musicians ("musical" people) are people who *want merely to play* and do play (or sing) for their own or others' satisfaction, and sometimes receive payment therefor. They cannot be blamed for being bored by numbers which *in no way improve their playing*, if playing and hearing music is their only serious interest in music. If by accident this book falls into the hands of mere performers, I not only am sorry for them, but use this opportunity for warning them against reading it. It is not meant for ordinary reading. It is a manual of practical exercises, very much like a multiplication table. Nobody reads the multiplication table. It does not help either the player of an instrument or the singer to play or sing with greater success.

Fourth, in the past the existing methods of teaching the use of number symbols for tone relations in complete pieces of music (thus going far beyond mere separate intervals between *two* tones) have been *abominable*, *very cumbersome*. *Learning* to use numbers, and *using* after having learnt it, both have

been drudgery pure and simple. I introduced some improvements into the use of numerical symbols twenty-five years ago. I have during these twenty-five years, off and on, either contemplating in my study or teaching in the class room, devoted considerable time to improving the method of familiarizing students with the use of numerical symbols. And I believe that the moment has come to try to overcome the horror of musicians. I hope and know from experience that those (young people, still eager to learn, young at least in spirit if not in years) who faithfully perform the exercises here offered and seriously study the explanations given, will convince themselves that the time which it cost them was well spent. They will gain a new insight into the essence of music through acquiring a new and powerful language,—a language in which music may be written and discussed with an exactitude which is as little limited by any "theory" of a particular historical origin or of any particular authority's dictation as the rules of arithmetic are thus limited. Let us have less of dogmatic and prohibitive "theory" and more "thinking" in the sense of knowing what we may and what we may not expect of music.

Really, why should musicians abhor and shun numbers? They ought to rejoice at the discovery that theirs is an art whose products are capable of far more exact description than the products of the painter or the dancer. That certain of the dynamic relations of the world, those represented by tones, *can be represented symbolically by mathematical ratios only*, and not in the poet's verbiage, is a fact to rejoice at not because of the "only", but just because they *can* be written down in such terms, which are absolute in exactness. Let us therefore proceed to learn how music can be translated from the written language of the player into the written language of the *thinker*, of the *critical* (mathematical) composer, of the *scientific* interpreter.

This written language, the musician's arithmetic, has one feature which ought to be very comforting to those who want to or have to learn it: The numbers which will be used in, or will result from, the musician's computations are *never very* large. And again: the very numbers which appear *most* frequently in his computations are the small numbers consisting of a *single* symbol, that is, the digits. The larger the numbers, the more rarely they turn up to bother him. Between 100 and 1000 he encounters only a dozen or two, and even these only on special occasions. With numbers above a thousand he scarcely needs to establish any acquaintance at all. The following pages of this treatise will prove this assertion.

Odd numbers almost alone, even numbers rarely, concern us. This exclusive use of odd numbers simplifies musical theory immensely

About two thousand or fifteen hundred years ago a discovery was made of truly revolutionary importance for the writing of music. It was the discovery that in the writing of music in arithmetical terms we usually may, and on many occasions for clearness' sake decidedly ought to, dispossess ourselves of

the number "2". In saying "the discovery was made" we must not adopt the notion that it was made suddenly by one man on one day and spread like wildfire over the world. It was undoubtedly made by many on many dates during many centuries. The fact historically consists only in the *universal* adoption more than a thousand years ago by all musicians of the western world of a method of writing which virtually banishes the number "2" from the musician's arithmetic. This signifies that in the present treatise we shall find it possible to eliminate all the even numbers, retaining only the odd numbers.

By the way it is worth recalling that in the earlier civilizations, those of the Orient, the Chinese, for example, the same discovery was made a few thousand years sooner. But when it was made later by the ancient Greek musicians, it seems to have been made by them quite independently of the Orient.

The preceding paragraph speaking of a discovery appears a little mysterious to him who takes literally what he reads. It is true that in the history of music we do not read of any banishment of the number "2" having been decreed in the earliest Middle Ages or just before. What is meant is the following historical fact. Those musicians of ancient Greece who preceded this discovery, when writing the chief "scale" of their music, named the tones from the lowest actually used to the highest by using the letters of the alphabet, A B C D E F G H I J K etc., except that for convenience' sake we write in this illustration the Roman instead of the Greek letters. Since their instruments had no great range and their singing practice no greater one, a considerable part of the alphabet was sufficient, the whole alphabet was not needed to furnish all the names wanted for all the tones used. The "discovery" then consisted in becoming aware of the fact that the same letter of the alphabet might, or even *should* preferably, be used for any tones which (being "octaves") *could be interchanged without making the melody unrecognizable*, interchanged without destroying the melody's identity in the judgment of any musician of good, practical sense and knowledge of the legitimate requirements of a real performance of his art.

In musical history the banishment of the number "2" is equivalent to the banishment of all the letters from the musical alphabet which follow upon the first seven, A B C D E F G. This event might very well be referred to in still another, a third phraseology, as "the discovery of the octave."

The musicians who "discovered the octave" did not know, of course, that this meant that they were banishing the number "2", for they were not thinking in numbers as we shall do here. But we now know it, and have known it for three hundred years. The vibration rates of such "interchangeable" tones are always of such ratios as 1:2 or 2:4 or 4:8 or 1:4 or 2:8, etc. The mathematicians have a single name for all those numbers. They call them "powers

of 2" and write for them the symbol 2^n, pronounced "two-high-n". The large symbol (2 in this case) is called the base of the power and the small and raised symbol (n in this case) is called the exponent of the power. The powers of 2 are, in regular series, 1, 2, 4, 8, 16, 32, 64, 128, etc. etc.

When early musicians "discovered the octave" and called several tones, lower and higher ones, by the same name, say, G, they were dealing with tones whose vibration rates, as we nowadays know, differed as 1, 2, 4, 8, 16 etc. differ. For example, if one of those tones had (might have been found with modern instruments to have) a physical vibration rate of 716, another had 358 vibrations and another had 179. Since *the identity of naming* in all such cases where the ratio is characterized by a power of 2, has been continually supported by the approval of more than a thousand years of musical practice, *common sense demands* that in our musical arithmetic we do likewise and refer to three tones, say, of the vibration rates 179, 358 (2 x 179) and 716 (4 x 179) by one identical number, 179, banishing the factors 1, 2, and 4 because they are nothing but "powers of 2" (speaking to mathematicians we should use a slightly different jargon and say "regarding all factors which are powers of 2 as representable by 2^0, that is, by the factor 1.") In this book, therefore, henceforth, every even number standing for any tone will have to submit to being divided by 2, and again by 2, and so forth until it can no longer be divided by 2, and to being replaced and represented by the last factor thus resulting.

This identification of tones and the resulting simplification of naming, used now for a thousand years past and recommended by us to the future, in the musician's calculation, is equivalent, we said, to the discovery of the "octave". Why octave? What a strange name for eliminating powers of 2 from the musician's arithmetic!

It is indeed a strange name, "the eighth" without nowadays being the eighth, a mere reminder of a remote past, analogous to "December", i. e., the tenth, which is a mere reminder of a past when people did not make up the calendar of solar years, but only of months (moons) in groups of ten. As December is an anachronism, so is octave. But it had sense in the remote past. At that time when early musicians of the western world, the Roman Empire, discovered *the melodic interchangeability* of some of their scale tones, the letters *left* after identification happened to be *only seven*, A B C D E F G. By historical accident such a scale was the most popular *among them*. The first tone identified *by them* with a previous one in the series, was therefore *the eighth*, again called A.

Thus it came about that *all identifiable* tones were called and are now called *octave tones*. Musicians of a different nation, using a more numerous scale or a shorter scale, would have made us familiar with another name than octave for two tones whose physical vibration ratio is a "power of 2".

"The discovery of the octave" is undoubtedly the greatest event which ever took place or ever could have taken place in the history of "musical

civilization." It is almost comparable in revolutionary importance to the discovery of fire in the history of general human civilization. Virtually everything we shall have to say in this book would be impossible without some implied reference to "the octave", that is, to the identifiability of tones "differing only by a factor which is a pure power of 2" in virtually all theoretical problems and in the great majority of practical problems of the player or the composer.

What are equally tempered scales? What is a convenient unit for measuring intervals?

At the present time the popular instrument is the piano. We do not use here the word "popular" merely in the sense of fashionable. The piano with its "twelve equally tempered tones per octave" *governs all our music*. Although the singer and the violinist are not by the nature of their "instruments" restricted to any particular scales, but could use scales of other (more than these twelve) tones if they wanted to, they simply do not want to because the piano has become their standard, too. It may be that the modern methods of making music on the phonograph or the radio will gradually make piano playing by non-professionals quite unfashionable and rare, or that the music of the future will feel the influence of a great composer's genius and thereby will free itself from the tempered twelve-tone piano, will transform this piano which has ruled us since the time of Bach into a somewhat different instrument. We shall have to demonstrate later *the possibility* (very limited possibility) of such an evolution. At present the twelve-tone piano is still the despot to whose tyranny the musical world bows.

What then is arithmetically an equally tempered twelve-tone scale? This question is not merely the result of a mathematician's curiosity. Its answer is a necessity for him who wants to comprehend the threefold historical aspect of music, that is, the past evolution of, the future possibility of, and the limitations by nature placed upon musical composition. It is often necessary to compare a certain interval in one (heard or only imagined) scale with another interval in the same scale or with a certain interval of another scale. That is, it is often necessary to ask and answer the question: "Which of these two, seemingly not very different, intervals is the larger one, and how much larger is it? As everywhere in quantitative comparison, so here: No such question can be answered unless we agree first on a standard unit of measurement.

The choice of a unit is a purely practical problem. In no science does such a choice necessarily involve a theoretical conviction. There is not, and ought never to be, anything speculative about it. The chief scientific requirement is that the unit chosen, if lost, should be replaceable. That is why the metric system (originally) chose as its fundamental unit a definite fraction of an earth-meridian. The earth is not likely to crumble away.

The musician, being a practical man guided by common sense, will

at once suggest that we choose as our unit the interval of an octave. That is most easily reproducible under the guidance of the ear if we should have no other aids. The human appreciation of the octave is unlikely to "crumble away." So far, so good. But there is in music the same objection to the choice of this unit which there was to using a whole meridian in the metric system of physics. The unit is inconveniently large. We therefore choose a definite fraction of it. Being accustomed to twelve so-called semitones per octave, we choose one-twelfth of the octave. This is a *convenient* fraction. It is not otherwise honorable or meritorious.

If an octave interval were, arithmetically, a definite difference of vibration rates which arithmetically added to that of any lower tone gave the higher octave tone, anybody would now know how to proceed. Everybody would divide that difference characterising the octave into twelve equal parts.

However, an octave is not definable as a *difference* of numbers. It is definable as a *ratio*. *Dividing* an interval represented by *a ratio* is by no means the same as dividing (fractioning) a common fraction. Dividing (fractioning) the octave interval which is represented by the *ratio* 1:2 is not the same as identifying that ratio with a common fraction, with one-half, and then dividing (fractioning) that fraction by the number (of smaller intervals) wanted, say, ten, and taking the resulting fraction for the *ratio* of the smaller musical interval wanted. For example, in a scientific sense it would be perfectly foolish to divide one-half by ten, getting one-twentieth, and then saying that the ratio 1:20 is one-tenth of an octave. Imagine an organ tuner *adding together* ten intervals of the ratio 1:20. The human ear would *not* recognize the big resulting interval as being *identical in span to an octave*, for it would be more than forty octaves. Even a single interval 1:20 would far surpass the span of an octave; it would surpass four octaves, 1:2:4:8:16:20.

In order to establish a unit of measurement for a musician's arithmetic, it is necessary to employ *logarithms*, because thus we *translate* (in mathematical language) *ratios into differences*. We have to find the logarithms of the terms of the ratio and to *subtract* one logarithm from the other. *This difference* (or any fraction of it) can become our unit of measurement.

Now let us do that for the sake of giving an illustration, fractioning the octave, that is, the interval of the ratio 1:2. In any (only four-place, if you wish) table of logarithms we find log 2 = 0.3010 and log 1 = zero. Subtracting this value, zero, we get as the difference 0.3010.

We divide this value (logarithm) by twelve and have the value 0.0251, again a logarithm. This latter value is our unit of measurement.

A better approximation is 0.0250858. Let us remember the method by which we found this unit of measurement. Let us remember that this is the measure of a semitone, the measure of S.

The *ratio* of the vibration rates *of this unit interval* is found by looking up in the table the number corresponding to the logarithm which itself is 0.0251. The corresponding ratio is thus found to be 1:1.059, or, if you shun the sight of decimal points, 1000:1059. That is, if the lower tone has 1000 vibrations and the upper has 1059, you have the interval of a true (true enough!) chromatic semitone. But it is not necessary to memorize this ratio. We can get along in everything this book tries to teach us without memorizing it. However, notice that adding 12 times 59, or 708, to 1000 yields only 1708, not the octave "2000". Remember that an octave is a ratio, not a difference, and is obtained *by multiplication*, using a constant factor, 1.059 *twelve times* repeated, or 1.059^{12}, giving 2.000, *not* by addition using a constant addend, $\frac{59}{1000}$ *twelve times added.*

At this moment a musician who has been watching us and has become interested in the conversation, approaches us and, although a stranger, asks: "Why did you, in order to reduce your inconveniently large unit, divide by 12? Why did you choose as your unit just one-twelfth?" I give him the following answer: "It is not I who chose 12. I merely accepted the choice made by millions and millions of people who, experimenting for more than 2000 years, reached a consensus of opinion. The result is your own piano. Examine it. Count the keys. Better music can be made by dividing the octave into equal parts, and into *twelve* equal parts, than by dividing it into unequal or into *any other number of equal* parts. I accept the practical unit found by mankind as my practical unit here."

"But", remarks the stranger, "have you not heard that it is the practice among the Siamese and some other Asiatics to divide the octave on their instruments into seven equal parts?" To this I answer: "Compared with the amount of experimenting done by all the European races, the search of the Siamese, in their practice, for the best manner of making music seems to have been limited. I rely rather on the results of the more varied European experiments in the performance of music. Therefore I use one-twelfth of 0.3010 rather than one-seventh of it. The choice of twelve for establishing a unit of measurement, however,—it should be remembered—is a *purely practical* one, neither a deduction from metaphysics nor a deduction from natural science. It is a mere convenience."

"Very well," continues the stranger, "then I am curious to know how large a Siamese unit is in comparison with the European unit." The answer is easy. One-seventh of 0.3010 is 0.0430. This is the measure of the Siamese unit. The measure of our unit we found to be less, 0.0251. Now, 0.0430 divided by 0.0251 is 1.71. That is, the unit interval of the Siamese is one and seventy-one hundredths of our unit interval, of the equally tempered semitone (so-called).

(A briefer solution of this problem is given below, for good reasons not here!)

Our attention has just been called to the fact that, whenever we want to compare intervals by reference to the "semitone unit," decimals make their appearance. Even the semitone, substituted for the "octave unit" that we found inconveniently large, is much too large in most instances unless we have no objection to using *decimal fractions of it* in such instances. About fifty years ago A. J. Ellis recommended that we get rid of decimal points by using in such instances a still smaller unit, which he called a "cent" because it is one-hundredth of a semi-tone. The measure of a cent is therefore obtained through dividing the logarithm of 2, i. e., 0.3010, by 1200, or dividing 0.0251 by 100. It is 0.000251. This logarithm is an exceedingly useful unit. And in the following pages we shall express it by the symbol ct, whereas the equally tempered semitone will be expressed by the symbol S. It goes without saying that S equals 100ct. Thus avoiding decimal fractions, we simply say of the Siamese unit interval that it equals 171 cents, whereas our own piano unit interval equals 100 cents.

Recommendation of a (modified) German nomenclature for scale tones

It is time now to give the reader of this "Arithmetic" a chance to acquire skill in the solving of problems by offering him a few for solution. But having just introduced him into a *terminology* (S and ct as numerical values) we might risk a little more talk about *terminology* before giving him the problems to solve.

We need twelve easily pronounceable and easily printable names for the twelve equally tempered tones of the octave. For seven of them, played on the white piano keys, we have the first seven letters of the alphabet. Nothing else could be better. But for the other five, played on the black keys, the nations speaking English as well as those speaking Romance languages have very cumbersome verbal expressions. They needlessly increase the number of syllables to be pronounced and talk of "flat, sharp, bémol, dièse, sostenido, etc." They require the printer either to set these words up in type or to provide himself with special type symbols to take their place. The nations of central Europe have for centuries used a much simpler designation. They simply add "is" to the alphabet letter when they mean, not that white key, of the piano, but the black key just following on the right hand side. Thus they say cis, dis, fis, gis, ais instead of c-sharp, d-sharp, f-sharp, g-sharp, a-sharp. Since English speaking people in other affairs have already established the reputation of being practical-minded and endowed with common sense, we shall prove to the world that they are capable of common sense in this "musician's arithmetic," too. Henceforth, throughout this book, the names for the twelve *keys* (and of course for the twelve *equally tempered tones*) will be this series: A, Ais, B, C, Cis, D, Dis, E, F, Fis, G, Gis. Not the slowest

to approve will be the printer, when we thus ask him to adopt something so easily printed as this language of Mozart, Beethoven and Grieg. (We refuse to follow Mozart in so far as he called the tone just below C, H instead of B.)

(Sample) Problem 1: Imagine the white keys of an organ tuned quite out of the ordinary, for a Siamese orchestra, in its equally tempered seven-tone scale; and call these seven keys by the small letters "a" to "g". Call the twelve keys of our common organ by the capitalized names enumerated in the preceding paragraph. *Identify the tone "a" with the tone "A".* Make an exact comparison of the tone "b" with the neighboring tones (Ais and B) of our common organ.

Solution: The measure of a Siamese unit is one-seventh of the octave. The measure of our common unit (S) is one-twelfth of the same span, the octave. Call the octave span Oc. An Oc equals 12S, and also equals 1200 ct. In order to know how many times the latter (European) unit (Oc:12) is contained in the former (Siamese) unit (Oc:7), we have to divide of two fractions one by the other, both of which have the same numerator, Oc.

Oc therefore disappears from the computed ratio

$$\frac{\text{Oc} \times 12}{7 \times \text{Oc}}$$

and the result is 12:7, which equals 1.71. That is, the interval from "a" to "b" is 1.71 times as large as S (a semitone) or 171 times as large as ct (a cent). The interval from A to Ais is 100 cents and the interval from Ais to B again 100 cents. Since the tone "a" and the tone A are identical, the Siamese tone "b" is located 71 cents above Ais and, of course, 29 cents below B. This is the answer.

(Sample) Problem 2: In the same case make an exact comparison between the Siamese "f" and our neighboring tones F and Fis. Identify again the tone "a" with the tone "A".

Solution: From "a" to "f" is a span of five-sevenths of an octave. We want to know how many times S (that is, Oc:12) is contained therein. The answer is found in the ratio

$$\frac{5 \times \text{Oc}}{7} \cdot \frac{\text{Oc}}{12} \quad \text{or} \quad \frac{5 \times \text{Oc} \times 12}{7 \times \text{Oc}}$$

and the result is 60:7, which equals 8.57 S, that is, 857 cents. The interval from "a" (or A) to "f" is located 57 ct above F and 43 ct below Fis.

Problem 3: In the same case locate the tone "c". Answer:

Problem 4: In the same case locate the tone "d". Answer:

Problem 5: In the same case locate the tone "e". Answer:

Problem 6: In the same case locate the tone "g". Answer:

To *make a graphical representation* of the six answers to the six problems which the student has just solved, is not only a good exercise which will give

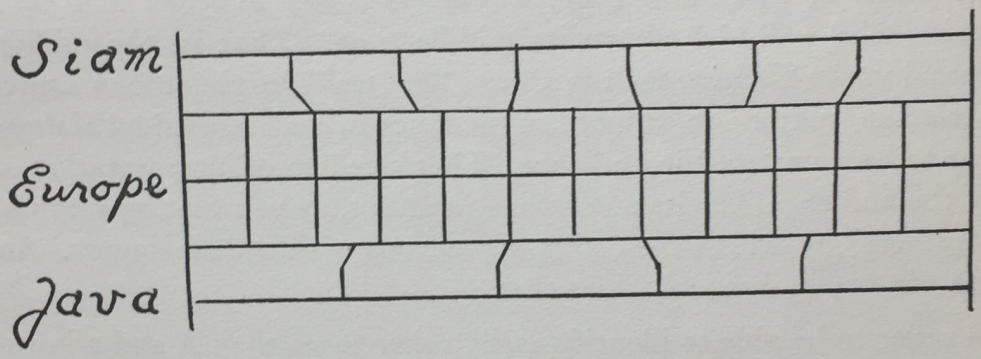
skill in a useful art, but is the only means of getting a clear insight into the relations existing between the effects of Siamese music on the audience and those of our music. Proceed as follows. Cut from co-ordinate paper which has inches divided into tenths, a strip three or more inches high and over twelve inches long. (Paste two pieces together if you have no sufficient length.) Then draw on it two parallel lines twelve inches long and one inch apart. Mark by vertical lines and by lettering the thirteen tones, A, Ais, B, C, Cis, D, Dis, E, F, Fis, G, Gis, A, on one line, and the eight tones from "a" to "a" on the parallel. You can now convince yourself that every triad on the Siamese scale is *neither* a major *nor* a minor triad. You recall readily, by looking at your key-board or the parallel line on your graph representing it, that any major triad (say, A-Cis-E) has three S below and four S above. But every Siamese triad you notice (say, a-c-e) is inevitably neither a Major nor a Minor triad, but must be called "a Neutral triad", for it has, in its total span of seven S, three and (approximately) *a half S* below and three and (approximately) *a half S* above its middle tone. That is, the middle tone of the triad in the Siamese scale is always "neutral", leaving the interpretation of the triad as "major" or "minor" to the fancy of each individual Siamese who happens to be a part of the audience. That freedom of interpretation, on the part of the audience, has its advantage (and not, as you first may think, only disadvantages), provided the composer reckons with the ambiguity. And the Siamese composer obviously does that.

In order to be able to identify easily other tones than A and a in the just mentioned graph you have made, you may *cut with the scissors* along in the middle between the two parallel lines, and then *slide them* relative to each other as you wish.

This will not be the last occasion on which we meet "half an S". Because half-S's will be forcefully brought to our attention, it is best here at once to get familiar with a better term for "half an S". These fractions of an octave (obviously twenty-four to the octave) are customarily named "quartertones". You could have said that you noticed the Siamese triad to be made up of a span of seven quartertones below and a span of seven quartertones above the middle tone; but you can make your statement more exact. Quite exactly, the Siamese triad has a total span of 686 cents, composed of two halves of 343 cents each. And a quartertone equals 50 cents. But in practice it is a sufficient approximation to speak, as we did, of 7 Q (quartertones) although only 350 ct (7 more than 343) are strictly equivalent to 7 Q, and only 700 ct equivalent to 14 Q.

In ordinary musical parlance you can say that the Siamese *fifth* is always nearly what you expect (the deviation being only 14 ct), but a Siamese *third* is never decisive, is always ambiguous, neutral (deviating 43 ct from the minor third and 57 ct from the major third).

Historically known human society has hitherto employed *only three tempered scales*, the wide-spread twelve-tone scale, the tempered seven-tone scale of the Siamese, and the tempered five-tone scale of the Javanese, of which we shall say a few words in the following paragraph. *There have never been any other* equally tempered scales. In the last two paragraphs the possibility was hinted at of understanding, merely for our convenience in comparing, the Siamese scale *as if* the following had happened,—as if the musician had *intended to use the twelve-tone scale*, but had done several things in order to need *only seven strings* (compare the graph, "Siam, Europe, Java"): thrown out the second tone (Ais) and substituted for the fourth and fifth (C and Cis) a "neutral" tone; then thrown out the seventh (Dis) and substituted for the ninth and tenth (F and Fis) a "neutral" tone, in the center between them; then thrown out the twelfth (Gis) and accepted again the succeeding tone (A) which starts the same octave cycle.



A SEVEN-TONE, A TWELVE-TONE AND A FIVE-TONE SCALE, ALL TEMPERED

Our future music is unlikely to make use of such ambiguities, i. e., of "neutral" tones. Does the reader believe otherwise? We can and shall have to regard *the universality of the twelve-tone scale* as established for the whole musical world of the future. Very slight *additions may be made* to these twelve tones—we shall later see how—but *no musical revolution* will ever destroy the twelve-tone scale as the universal foundation of all future music. This conviction of the present writer is quite contrary to the teaching of Julian Carrillo,[1] who is not only strongly convinced that "a musical revolution" is impending, but also considers himself as the prophet ushering in this revolution in a similar way as the "encyclopedists", Diderot and so forth, ushered in the French Revolution.

The musicians of Java have *divided the octave into five* equally tempered intervals. The tones from octave to octave tone then are located at the following points, measuring the distances in cents: 0, 240, 480, 720, 960, 1200. It is obvious that this five-tone scale, for our convenience in making the comparison, can be regarded as if the musician had *intended to use our twelve-tone scale*,

1 For example, in his propaganda in Mexican newspapers. See Appendix 7.

but had done the following things in order to need only five strings (compare the graph again): had thrown out Ais and substituted for B and C an intermediate tone; thrown out Cis but used D, thrown out Dis but used E, thrown out F and substituted for Fis and G an intermediate tone; and had used A.[2]

Biology, not metaphysics, restricts us to a rather small group of the smallest prime numbers

We remember now "the discovery of the octave", made nearly two thousand years ago, which means the elimination from our present musical arithmetic of all factors which are pure "powers of 2". That is, if we come across numbers like 280 or 144, we preferably do not write such (reducible, but unreduced) numbers at all, but divide by two, and again by two, and so forth. And thus instead of 280 we write 35 and instead of 144 we write 9. And then we discuss the musical relations between 9 and 35.

A second discovery, equally ancient, is this: of the series of prime numbers (what "prime" numbers are, any dictionary will tell him whose memory it has escaped), that is of this series 1, $(2^n)^3$, 3, 5, 7, 11, 13, 17, 23, 29, etc., only the very smallest ones (perhaps only 1, 3, 5, 7) enter as factors into the arithmetic of the musician. Even 11 is probably already outside of our sphere of interest. This fact has always been gratifying, but it has also been puzzling, until the beginning of the twentieth century. Only *contemporary biological science* has made it clear that (1) not imaginary metaphysical entities, but the conductive properties of *nerve* fibres must be held responsible for this fact, (2) the conduction of nervous excitation is a *chemical* phenomenon, (3) chemical compounds are geometrical configurations where *the prime numbers* by necessity enter as factors the more conspicuously the smaller they are (any glance at chemical formulas makes that clear).

For that reason even 11 is most probably already outside of our sphere of interest, as being too large a prime number; and if we ever again mention this prime number, the reader should take it almost for granted that it is for curiosity's sake only. But numbers like 15, 21, 35, although they are larger

2 The student who reads this book for the first time had better pass over any further consideration of Javanese music at this moment. It would confuse him. To the student who reads this book for the second time and has thoroughly mastered its contents, I suggest the following drill problem: If the five tones happen to be interpreted by a member of a Javanese audience as 3-7-9-1-21-3, how much would each tone as interpreted *differ in cents* from the tone as played, provided the tone symbol "3" is identified with the position of zero cents or 1200 cents? Of course, we suggest this as a drill problem, and not in order to foster the assertion that the Javanese audience could not interpret such music *also in many other ways*.

3 The prime number 2 does not really belong in this series on account of what has been said about the "discovery of the octave." Therefore it appears in parenthesis. In the discussions with which we are now concerned, all the powers of 2 are already symbolically represented by the prime number 1 in the series appearing above.

than 11, will concern us, because they are products and their factors are smaller prime numbers than 11.

The great practical problem for the student of "a musician's arithmetic" consists in *learning to talk of ratios* whose terms contain the prime numbers 1, 3, 5, and 7 (and, as we just saw, these prime numbers exclusively) as factors, in the numerators and denominators of the fractions expressing the ratios. ("2" is omitted from the list of prime numbers because, we remember, in "discovering the octave" we have dispossessed ourselves of all even numbers; "11" is omitted because a prime number as large as that is unlikely to be needed, but whoever wants an intellectual chastisement may put it back.)

The student cannot talk intelligently of ratios unless they mean something to his ear. Unfortunately he has never met a teacher (have you met one?) capable of training him to connect melodic phrases with ratios of numbers. The ratio 1:2 is the only exception. And he *discovered* that only in order to *disregard* it, not in order to use it positively. He *was easily trained to dispossess himself* of the prime number "2" by discovering the octave and, in consequence, by *merely repeating* the letters after the first *seven of the alphabet*. But he *never was trained to seize as his property* the other prime numbers because seizing the other prime numbers would involve much more than mere juggling a few more alphabet letters then seven ever did involve; it requires systematic training in *a special kind of arithmetic*. He must be trained first to simplify all those horrid looking things called numbers which he will meet, by reducing them to *odd numbers*, then to analyze what remains into *factors* of 3, 5, and 7, and, last but not least, to connect *melodic phrases* with ratios built out of 1, 3, 5 and 7.

Now we must give the music student that training which he has hitherto lacked in the notation of music by numbers,—the numbers 1, 3, 5 and 7 and a few of their powers and products, or—leave him forever an intellectual slave stultified by the "theories" (which, like the ready-made clothes suggested by Henry Ford, come in one size only) of historical "authorities" who *pretend* to relieve him of the burden of thinking for himself.

By looking in the index under "Rameau," the reader will discover the page in this book on which there is a quotation indicating what is meant in the preceding paragraph by "authorities who relieve us of all thinking" beyond that *ready-made by them for all our occasions*, tending to put a stop to all musical invention.

The construction of a "table of standard spans." The utilization of the table in solving some rather gross problems of the composer

(Sample) Problem 7: Let us represent any pure power of 2 by the symbol 1. State in units of S and ct all the "spans" between two tones of which one is represented (as a ratio term) by a pure power of 2 (that is, by the symbol "1")

and the other by the product formed by multiplying 3 with a pure power of 2 (that is, by the symbol "3").

Solution: *Imagine* that the vibration rate of the tone expressed by the symbol "3" is actually 3, (that is, by good fortune not 6 or 12 or 24 etc.), and that the other tone (symbolically "1") is *higher*, but less than an octave higher than 3. The actual vibration rate of the other tone can then be nothing else than 4. If it must be a power of 2, we can choose only from 1, 2, 4, 8, 16, etc. If it must be a number greater than 3, since it has just been assumed to be higher, and if it must be less than 6 since it is less than double 3 (the octave) that would leave only 4 for our present consideration.

Now proceed in a way analogous to that which on a previous page gave you a measure of the octave span. In a table of logarithms we find log 4 = 0.6021 and log 3 = 0.4771. In order to get a measure of the span between these two tones, the latter number is to be subtracted from the former. This subtraction leaves 0.1250. How many times is S, that is, we remember, 0.0251, contained in the last-mentioned number? The answer is 4.98 times. Therefore, the span between tone "3" and the higher "1" is 498 cents or 4.98 S.

Accordingly, the span from the same "3" *to the second higher* "1" is 12S additional, that is, 16.98S. The span from "3" must make a total of an octave, 1200 cents, and you will admit that 702ct (from "3" *to a lower* "1") *added to* 498ct (from this same "1" to the lower "3") *does make* 1200 ct. Still one more span let us expressly mention: the span from "3" *to the second lower* "1" is 12S added to 7.02S, that is, 19.02S. All the other spans called for in the problem are obtainable by the further addition of octave spans, that is, of multiples of 12S. Reader, give examples:

Same Problem 7. Another solution, half given as a sample, but *to be finished by the student:* If we now take the tone "3" to be the higher and "1" to be the lower one, with the span of less than an octave, the actual vibration rate of "1" should be taken as the *next* to 3 lying "power of 2" which also is less than 3, so that "3" *can be* the higher tone. The lower tone of course, has a lesser vibration rate. That is, you solve the problem by taking the higher tone "3" again as actual 3, but the lower tone "1" now as actual 2 (since the real number 1 is *not next* to 3 in the common, here backward, counting series and 4, 8, 16 etc. are *more* than 3). Therefore, find the logarithm of 3 and the logarithm of 2, subtract the latter from the former, divide the remainder by the measure of S, tell us the result in cents and compare it with the preceding solution of problem 7. Answer: . . . . divided by 0.0251 equals . . . ..

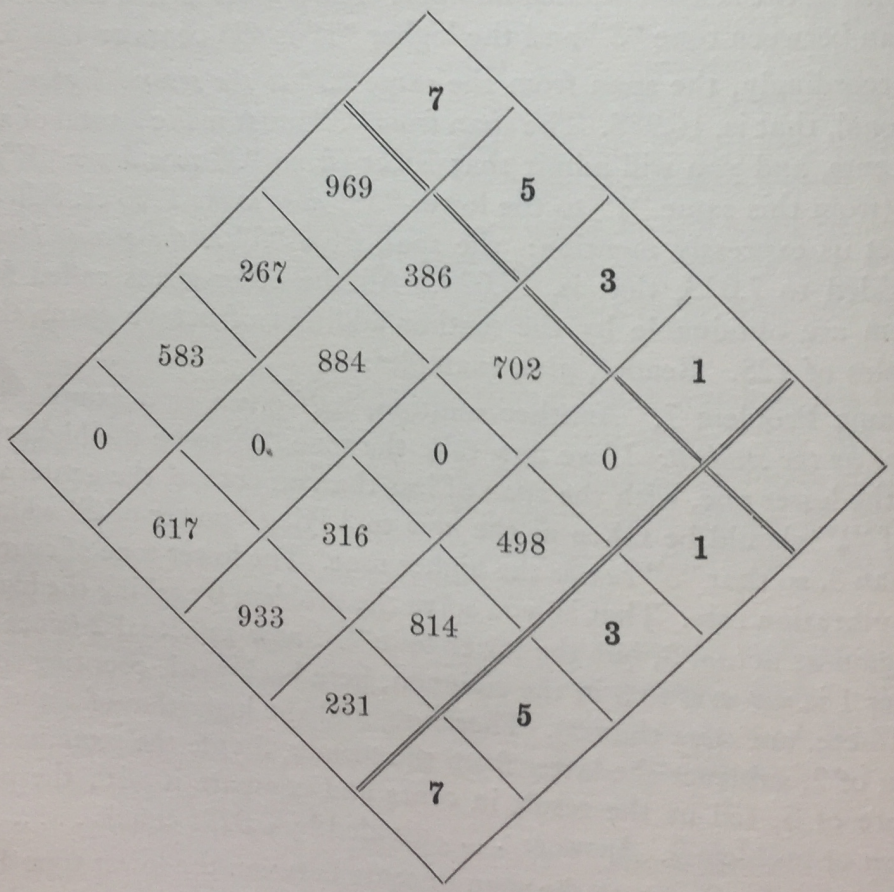
Problem 8: Compute the span in cents between the lower tone 4 and the upper tone 5. Since you may not have any table of logarithms handy, we tell you that the logarithm of 5 is 0.6990. Answer:

Problem 9: Compute the span in cents between the lower tone 4 and the upper tone 7. We give you the logarithm of 7 as 0.8451. Answer:

Now look at the table which has the shape of a square, standing on a corner. Discover within it the numbers which you have computed as the answers of the problems 7, 8, and 9. Invent a way of reading off from this table the answers to any problems similar to the seventh, eighth, and ninth, which might be given you in the future, so that you will then be able to save the time it would take you to compute the answer. Which of the *two rows of black-faced numbers* contains the lower tone of such a problem? And which row contains the upper tone? Further, how do your searching eyes have to wander until they find the number of cents for the span?

Problem 10: Can the numbers within the table be so paired that each pair gives exactly the same sum? What is that sum? How are the numbers located that do pair in that way? And why do the pairs give that particular sum and not a different sum? Answers:

Problem 11: Compute the span in cents between the lower tone 8 and



"SPANS" IN CENTS BETWEEN THE COMBINATIONS OF THE TONES SYMBOLIZED BY 1, 3, 5 AND 7 WITHIN *ONE* OCTAVE. BEYOND ONE OCTAVE, ADD MULTIPLES OF "1200 CT."

the upper tone 11. The logarithm of 8 is 0.90309 and the logarithm of 11 is 1.04139. Answer:

A bit of general psychology applied to hearing—real or illusive

Before the next problem will be comprehensible to you, it may be necessary to remember an analogy taken from language. The Chinese in speaking do not use either our R-sound or our L-sound but a sound resembling both about equally, *intermediate* between the two. This has a peculiar effect. When we ask a Chinaman to say "glue" and therefore expect to hear "glue", *we think* that we hear him say "grew". And when we expect him to say "grew", *we think* we hear "glue". Likewise, instead of "red" he seems to say "led" and instead of "led" he seems to say "red." But when we ask him to say "university", he seems to give us the right sound, because, it so happens, we have no word (and therefore do not ever expect to hear) "univelsity," although he does not say "university" with a distinct "r". Or, when the people in London say "hair", we think they say "air", and when they say "air" we think they say "hair". All of this because *we* usually make a strong *distinction* and *they* usually pronounce an *intermediate* sound. In general there is no need of everybody on earth who speaks the same language (say, English) having exactly the same pronunciation of words. Mere similarity is sufficient in order that we "hear" what the other fellow wants us to hear, provided his actual sound is *not just intermediate*, that is, provided it comes nearer to one of two possible sounds of different meanings *than it does to the other sound*. That intermediate pronunciation of course is ambiguous. Now you will surely comprehend the following problem.

Problem 12: Imagine that you are making music to a person who has a perfect conception of how both the intervals 2:3 and 8:11 ideally sound. (To know the latter tone relation, 8:11, he would have to be wonderfully gifted,—but that improbability does not matter here, in an imaginary problem.) Now you have nothing but your ordinary piano to make your music on. Let us agree on that assumption. Why would there be no difficulty, then, for you in making your audience understand that former interval when you give it? Why would the latter case (8:11), however, remain hopelessly misunderstood on your piano and therefore virtually impossible of execution thereon? Why this difference between 2:3 and 8:11? Derive the data which you need to make your answer plausible from problem 11 and from that previous table. *How* might 8:11 be *misunderstood* if you *insist on playing it* on your piano? That is, what might be understood in its place? Look at the table! The kind of misunderstanding which is most probable depends, of course, on how you *try* to play it on the piano. Answer:

Is your *practical* loss in your piano music of the interval 8:11 serious? In order to save the ratio 8:11 for your use, someone rashly suggested that the piano be improved (?) by building it with 18 keys per octave, and thus with

18 equally tempered tones in intervals of 66 ⅔ ct each, instead of the 12 you have now in intervals of 100 ct each. Before you submit to having your piano "monkeyed with" in *this* fashion, solve the following two problems:

Problem 13 (similar to problem 14): Imagine your audience so improbably gifted as the one in problem 12, knowing "by the ear" both the intervals 2:3 and 8:11. But imagine that you have no ordinary piano, but only that of the anonymous reformer just mentioned, with 18 equal key intervals to the octave. Which of the two intervals, 2:3 and 8:11, could your audience now *more readily* (with more certainty) be made to understand than the other? And what multiple of the 18 equal units would you use to play it? Answers:

Do the following figures have any relation to the answer to the last problem? (*Notice* that the first and the last number in both lines are *multiples of* 66⅔ *cents*.)

666⅔ ----------702----------733⅓
533⅓ ----------551----------600

Problem 14 (similar to problem 13): Prove that in a scale of 18 key intervals per octave on the piano, the succession of the first, eleventh, twelfth scale tones would have this *bewildering* effect on the hearer: He would after the first tone (first key, zero cents) hear a second tone (eleventh key, 10 x 66⅔ cents) and then a third tone (twelfth key, ? cents) distinctly different from the second and yet very likely having to the listener with respect to the first tone exactly the same meaning which the second had. What would be this identical meaning which the second had (that is, which of the ratios of the table above referred to)? Answer:

Such a bewildering effect does not deserve the name of a "musical" esthetic effect. That it may deserve the name of a "noisily" esthetic effect, a slide, a species of "jazz", need not be denied. There is a demand for the latter, too. By all means admit that.

In dignified music—music in the highest sense of the word—it is *almost* as important to play this particular interval, 2-3, with a *minimum* of deviation from the true, as it is to keep the octaves "pure". Even the Siamese recognize this, indeed could not without this purity use "the neutral third" in the way they do. They have only a *deviation* of about 14ct in this case, 2:3, while their smallest real *step* is twelve times as large, 171ct. Our inborn conception of the interval 2:3 is too strongly effective to permit the practical musician deviations of considerable magnitude.

That human beings have any, more or less, effective inborn conception of the interval 8:11 is hard to disprove. But it is easy to prove that (if it is not zero) it is immensely weaker than the inborn conception of the interval 2:3. The ratio 2:3 is next to the octave the greatest wonder of the sense of hearing. No proposal of a method of octave division in instrument building can afford to slight this ratio, although it might without any loss to music slight

the ratio 8:11 by making its use impossible, as on the piano. The next two paragraphs will further illustrate our inborn familiarity with the ratio 2:3.

In "The Story of Notation" (London, 1903) C. F. A. Williams says on page 65: "Many musicians heard in the late eighties the unemployed of London walking about the street and singing; the tenors pitched their voices at a Fifth (ratio 2:3) above the basses, and sang the tune with them quite happily. This was the ancient organum pure and simple." The ratio 2:3 is here employed in a manner in which more commonly the ratio 1:2 is employed.

And the following sentences translated from Carl Stumpf's "Tonpsychologie," (Leipzig, 1890) p. 179, tell the same story: "Singers devoid of any musical scholarship,—musically gifted savages as found here and there in the midst of our musical civilization,—not infrequently give us a chance to hear Parallel Fifths (ratio 2:3) intended to be unison (octaves). Mozart tells in his letters that he once was able to listen to it in Milan. I heard it, more conveniently, in my own servants' quarters."

It is almost incomprehensible, therefore, that anybody should ever have seriously proposed to divide the octave into eighteen equal parts and thus to sacrifice the purity of the ratio 2:3. And a musician, too!

But treatises proposing to give a "scientific" discussion of musical theory more frequently display vagueness than accuracy of thinking. That is as unfortunate as it is true.

Use of the table of spans in solving a few further and more delicate problems of the composer

We have been trying to make the student familiar with the arithmetic which enables him to judge what ratios of small numbers are very conveniently suggested to his ear by playing on his own piano made equally tempered by the tuner. "Very conveniently" means, of course, with the least deviation from the true, from that which is intended. Thus far the student has found the twelve-tone scale of the piano very suitable. But he has tested only the tone pairs symbolized in the terms "1, 3, 5 and 7." In problem 12 he added to them the term "11", testing its possible presentation on the piano in spite of doubt about the need of a prime number so high.

The *chemical* formulas determining the mutual *co-operation* of nervous functions in a brain, and therefore the *musical* effects of tone combinations, are far less likely to include the number 11 than the smaller prime numbers. The chemistry of the nervous system is still largely unknown, it is true. Aside from being held of *rather improbable value*, however, the number 11 should *not be dogmatically excluded* for *this* reason, nor for the *other* reason that, as we found, it is impossible of execution on the twelve-tone piano. Let us prove all things and hold fast that which is good, in theory as in practice.

Far more important, for our present discussion concerning the biological basis of the effects of music, than the prime number "11" are composite num-

bers which are products of the smaller prime numbers, such as 9, 15, 21, 25, 45 and others. They fit better into the molecular geometry of the chemists. In the invention of tunes, these numbers are likely therefore to play a role in addition to 1, 3, 5 and 7. And besides, most of the smaller numbers among them *are capable of execution on our common twelve-tone piano.*

The problem which now opens up before us is this: What is the *real meaning*, in numerical tone relations, of various, but typical, *simple tunes* when these tunes are played *on the equally tempered piano?*

The simple tunes which the student might suggest for such a computative analysis are likely to belong to numerous classes, that is, to numerous scales. Some will probably belong to the "pentatonic" scale, some to the "diatonic" scale, and some perhaps to various "more complicated" scales. Whether we have just mentioned *three types* or merely three among a lot of scales possible, is still a question, which will concern us later when we consider a possible basis for distinguishing true types of scales. Nevertheless, before we proceed to the solution of *further* problems in musical arithmetic, it is well to say a few words about the origin of these terms, "pentatonic" and "diatonic."

A "scale", of course, is the *traditional* name of any array, from low to high, of all the tones of which a tune makes use. Perhaps we should rather say of all the "notes" than of all the "tones," since (1) for more than a thousand years it has been customary to leave out tones which are mere "octaves" of others. (2) Besides, we should make a difference, to which musicians are quite unaccustomed, between "notes" in the sense of "key-symbols" which merely indicate what key to strike on the instrument and "notes" in the sense of "ratio terms." In making this distinction throughout our further discussions we display our great faith in the willingness of the more intellectual members of the profession to accept an unaccustomed mode of thinking.

Usually the notes of a "scale" are arranged so that the enumeration from low to high begins with that note which is most emphasized in the tune, that is, with the so-called key-note. We shall here entirely avoid the term "tonic" because that term has other connotations than "most emphasized in the tune" and is therefore ambiguous, confusing. This application of qualitative or intensive emphasis, alone, not to mention time values, rhythm, makes a considerable difference in appearance between a tune and its scale, for a tune does not always begin—nor end—with its most emphasized tone, its key-note. *A scale* however is *customarily* written beginning with *the most emphasized note*, the "key-note", and repeating it at the other end.

Pentatonic literally means nothing but "of five tones", that is, of five key-symbols. That is, of *five* provided we omit the repetition at the other end. In musical parlance *not all five-note scales* are included under "pentatonic",

but only the scale with the following succession of (tempered semitone) *intervals* going up from the key-note:

—2S—2S—3S—2S—3S

within the four pairs composed of the (five dashes) five *notes*, i. e., *key-symbols*. The 3S on the right indicates merely the interval we would find if we did repeat the key-note at the other end of the scale. Examples of such music are common among the folk tunes of many nations. Recall "Auld Lang Syne". You can write it in the key-symbols C D E G A if you wish.

Diatonic literally means "thorough-going". In musical parlance *it means nothing very thorough-going*, but simply an array of (semitone) intervals (the dashes will represent the tones!) going up as follows:

—2S—S—2S—2S—S—2S—2S

and called by us A B C D E F G, substituting for the seven (dashes) notes seven alphabetical key-symbols in imitation of the Greek practice. The history of the notation and interpretation of this scale is so interesting, helping us to get rid of various prejudices in musical interpretation, that it is treated in more detail, though briefly, in a special appendix of this book.

It is obvious to anyone who looks at both scales that the pentatonic scale is contained within the Greek diatonic scale. Indeed it is contained therein twice, as C D E G A and as F G A C D. Therefore it is best to study the (smaller) pentatonic scale first and the (larger) diatonic scale later.

The pentatonic scale is just conveniently small, simple, to serve for a student taking for the first time exercises in "the musician's arithmetic", without being too small, too simple, that is, without exposing him to the erroneous impression, or let us make it stronger and say "the common delusion," that a scale in numerical interpretation is *as simple as* the small number of "notes" in the sense of "key-symbols" might make him believe. *Five notes do not necessarily mean only five numbers*, that is, ratio terms. This is an extremely important fact to learn, to drill into oneself and to remember forever. Ignorance of this fact has filled many pages in the literature of musical theory with wasteful quarreling.

(Sample) Problem 15: Enumerate all the pairs of the pentatonic scale F G A C D which can plausibly represent the number symbol "1↗3" or "3↘1" (to be pronounced "1 going up to 3" or "3 going down to 1").

Solution: By referring to the previous table which has the form of a square standing on a corner, we observe that "1 going up to 3" is (if we agree to keep the tune within one octave) a span of 702 cents. On the piano any interval of 700 ct or seven S is therefore an excellent means of performance. Thus we quickly find the following pairs of notes being capable of representing the (melodic or harmonic) tone relation stated in problem 15: F—C, G—D, C—G and D—A. Our problem is solved.

Let us now utilize the solution of problem 15 and represent in number

symbols (in the smallest possible number symbols, of course) the whole pentatonic scale F G A C D as a unitary music system which requires, *as agreed on merely for the present*, that F—C, G—D, C—G and D—A be all identifiable with 1—3. Starting then by chance with F—C, we call F "1" and C "3". But now we are compromised and unable to call C—G "1—3", because C has already another name, that is, "3". C—G, therefore, is now called "3—9". For the same reason, being compromised on account of having just given G the name of "9", we call G—D "9—27." And D—A, using the same reason, we can not now help calling 27-81. (Remember that there is no virtue in having just F and not one of the other letters written on the *extreme left*. F is in that place only by chance.)

F — G — A — C — D
1 — 9 — 81 — 3 — 27

Problem 16: Enumerate all the pairs of the pentatonic scale F G A C D (remember that F is only by chance on the left!) which can represent the number symbol "3↗1" or "1↘3"; and then utilize the result and represent in number symbols (using the smallest possible numbers, of course) the whole pentatonic scale F G A C D again. Perhaps you will be surprised in comparing this music system with the one which resulted from problem 15.
Answer: (a) The pairs are:

(b) And the whole scale is:

Problem 17: Enumerate all the pairs of F G A C D which can represent the symbol "1—5" in accordance with the previous "table of spans". Answer:

Utilizing the solution of problem 17 and combining it with the solution of problems 15 and 16, we now have the pentatonic scale:

F — G — A — C — D
1 — 9 —5-81 — 3 — 27

(Sample) Problem 18: Compute in cents the span between the tones
(a) "5" and "81"; also between
(b) "9" and "5"; and between
(c) "81" and "3";
and give reasons why the *one tone* "A" of your piano may be "heard" as if it were "5" on one occasion, and "81" on another occasion.

Solution: (a) We have in our table (standing on point) "5" but not "81". Nevertheless we can use the table, for 81 is "3 times 3 times 3 times 3". The table tells us that going up from "5" to "3" is a step of 316 ct. Going up from "3" to "9" is the same as from "1" to "3" and therefore 702 ct. From "9" to "27" is again 702 ct. From "27" to "81" is again 702 ct. Adding these four steps we get a total of 2422 ct as equivalent to going up from "5" to "81". But since in our arithmetic "powers of 2" are usually neglected and steps of "1200 cents" are equivalent to "powers of 2", we neglect, that is, subtract from 2422

ct "1200 ct" (that is, 12 S) once and again. The remainder of 22 ct shows the upward step from "5" to "81" to be a trifle more than the fifth part (than 0.20) of a semitone, that is, a trifle more than ⅕ S.

Therefore we cannot but use the same piano tone for both "5" and "81" whenever either of these is wanted! Ignorance of this fact of possible double meaning of a single and definite piano tone was above spoken of as having caused a lot of wasted quarreling filling many printed pages in the literature of musical theorizing.

(b) Further, what is the upward step from 9 up to 5? The table does not contain "9" but it tells us, that from "5" up to "3" is 316 ct and that from "3" up to "9" is 702 ct. Therefore, from "5" up to "9" is (316+ 702) ct, from which we need not this time subtract any multiple of 1200 ct because this sum, 1018 ct, does not exceed the single octave span, 1200 ct. Since from "5" to the next "5" is self-evidently 1200 ct, and from "5" to "9" is 1018 ct, from "9" to that next higher "5" must be, by subtraction, equal to 182 ct, that is, almost 2 S. Or, we could have proceeded in this way: from "9" up to "3" is 498 ct, from "3" up to "1" again 498 ct, from "1" up to "5" 386 ct. Total of 498, 498, and 386 is 1382. Minus 1200 ct leaves 182 ct. As *several* roads lead to Rome, so to our goal here.

(c) Now the student will compute the step from "81" to "3" (in the tabular statement preceding sample problem 18) and complete the solution of this problem. Answer:

Further demonstrations of the fact that a definite group of physical tones (notes indicating strings, keys, organ pipes) serves, that is, on occasions and with individuals can serve, to make music of far greater groups of tones than indicated

The very great importance of the last problem lies in the insight which its solution gives us into the fact that this scale, *obviously of six tones in the psychological sense*, 1-9-5-81-3-27, is in the practical sense of the musician-performer nothing but the "pentatonic" scale, a scale of only five key-symbols. But we shall soon discover similar facts even more surprising, and at the same time comforting because they simplify the performance,—the facts which essentially make our piano and other instrumental music as easy as we know it in everyday life.

Problem 19: Enumerate all the pairs of F G A C D which can represent the symbol "1-7" in accordance with the previous "table of spans". Choose 7↗1 rather than 1↗7. Answer:

Utilizing the solution of problem 19 for F–G and C–D *only* (disregarding G–A, for reasons to be understood later) and combining it with the last number series we obtained for the pentatonic scale, that is, 1-9-5-81-3-27, we have now the following pentatonic scale:

$$\overbrace{F}^{63-1} - G - \overbrace{A}^{5-81} - \overbrace{C}^{189-3} - D$$
$$\overbrace{63-1}^{F} - 9 - \overbrace{5-81}^{A} - \overbrace{189-3}^{C} - 27$$

Here we have, in the psychological sense, *eight* tones, and yet in the practical sense of the musician, especially when performing on the piano or any other instrument with tuning fixed in advance of playing, there is here only a *pentatonic* scale, a scale of only *five* key-symbols.

Good common sense dictates, of course, the conviction, even though this conviction may not be provable, that the *higher numbers*, "63", "81", and "189" are much less likely to be those as which the meaning of the tones F, A, and C is "heard", than the *smaller numbers*, "1", "5", and "3". Surely, none of our readers will debate this. On another and later occasion arguments supporting this conviction will appear. At the present moment such arguments are better avoided because they would sidetrack us.

Let us say, then, that in addition to a certain most probable meaning, any scale can assume other and less and less probable meanings, all depending on the habitual meanings which that scale has acquired for the ear of the audience and also on the temporary preoccupation of the audience with this or that meaning. He who is prepared to hear something said, may hear it said even by the waves of the ocean, or the winds of the desert,—and not only in a poetical sense, but in a very realistic one.

The above statement is *a rule* so important for the musical practice of the whole world and of all epochs of history, so all-pervading, that the student will greatly profit from memorizing it.

Problem 20: Prove that the reason why in the paragraph directly following problem 19 no answer was utilized for G-A (but only for F-G and C-D) is a *double reason:* (1) that it is a purely practical reason and was partly contained in the three paragraphs preceding this, and (2) that it was partly contained in, or was at least analogous to, the solution of problem 12.

Answer 1: Answer 2:

Problem 21: Which of the following six, here arbitrarily presented, number symbols "29, 35, 107, 313, 567, 607" *cannot* possibly (if we accept as reasons the results of the discussion between problems 6 and 7) appear mentioned in the tabular scale display directly following problem 19, *nor in any* similar scale; and what is the simplest reason why they cannot? And which of the six for *these* reasons *could*, but for *another reason* mentioned more recently *do or does not* appear? Answers:

Problem 22: Enumerate all the pairs of F G A C D which can represent the symbol "3↗5" (pronounced "3 going up to 5"). Answer:

Problem 23: Why does the solution of problem 22 add *fewer* tones to our last "psychological" scale corresponding to the "pentatonic" scale of practice, than there are pairs (bracketed number symbols) in the answer to problem 22? Answer:

We now have the following pentatonic scale:

F	—	G	—	A	—	C	—	D
189-3	—	27	—	15-243	—	567-9		5-81

Problem 24: Why have the solutions of problems 22 and 23 caused the symbol "1" to disappear from our pentatonic scale, and why have all the other number symbols of F, G, etc., become *almost* (not really to the skilled observer) unrecognizable? Answer: Because a certain arithmetical operation was unavoidable, namely:

Since in the last numerical symbolization of the pentatonic scale many number symbols have become of formidable size, we may use the same argument which we have used before, just preceding the statement of problem 20. (What was that argument?) For most *practical* purposes, then, we could try to *forget* the existence of the three largest numbers (*but not thereby rule them out of existence*) and write our pentatonic scale now simply thus:

F	—	G	—	A	—	C	—	D
3	—	27	—	15	—	9	—	5-81

Problem 25: Enumerate all the pairs of F G A C D which can represent the symbol "3↗7" (pronounced "3 going up to 7"). Answer:

You will surely not add any new numbers to the pentatonic scale in consequence of the answer to the last problem. One of them that might be added is "189," which we had already within the scale and just omitted as being *probably* but rarely needed. The other number is "35," which we shall omit from the pentatonic scale because it is most likely be to misunderstood as "9". The following problem is intended to give the answer to the question "why thus misunderstood?"

Problem 26: Suppose A represents "15" exactly, in the scale

A	—	C
15	—	35-9

and C, of course, is 300 cents above A. How, then, using cents as units for measurement, is C located relative to "35" and "9"? Is it nearer to "35" or to "9"? Answer:

If A represents 15 and the composer decidedly wants a certain higher tone to be "heard" as 35, it might be advisable to add to the instrument a new string or key producing a lowered C, located about *half way between* C and B, that is, half an S below C.

This is not the first occasion on which we have met "half an S", and will not be the last; and because half-S's will again be brought forcefully to our attention, it is best, as was noted on page 17, to get familiar with a better term for "half an S." These fractions of an octave (obviously twenty-four to the octave) are customarily named "quartertones".

Practical use of the concept of quartertones

To repeat from page 17: You could have said that the Siamese triad is made up of a span of seven quartertones below and a span of seven quartertones above the middle tone; but you can make your statement more exact. Quite exactly, the Siamese triad has a span of 686 cents composed of two halves of 343 cents each. And a quartertone equals 50 cents. But in practice it is a sufficient approximation to speak of 7Q (quartertones) altho only 350ct (7 more than 343) are strictly equivalent to 7Q, and only 700ct equivalent to 14Q. In ordinary musical parlance you can say that the Siamese Fifth is always nearly what you expect, but a Siamese Third is never decisive, is always ambiguous, neutral.

Never forget that the variety of tones in a piece of music may greatly surpass the variety of key-symbols

Problem 27: Enumerate all the pairs of F G A C D which can represent the last pair of symbols, "5↗7", directly suggested by our table of spans. Answer:

Since the answer to the last problem does not offer any further addition to the numerical symbolization of the pentatonic scale, we have reached the following convictions:

(1) that "the pentatonic scale", if understood as a more-than-five-tone scale, is *likely* to be "understood" by the audience in accordance with the six symbols contained in this following series, provided we choose to enumerate the symbols, not as they succeed in the scale, but as they succeed from small to large in common "counting": 3, 5, 9, 15, 27, 81;

(2) that such higher numbers as, e. g., 189, 243, 567, and many others *may* also play a role in the "understanding" by the audience of what is played, but less probably;

(3) that "1" does not stand in the series of the first six only because among F G A C D we gave a double interpretation ("5" or "81"; compare what precedes problem 25) to D, but

(4) that we can choose to give D a single and A a double interpretation ("5" or "81", compare what follows problem 17) and then have to put down the first six (smallest) numbers as 1, 3, 5, 9, 27, 81, making no use of the number 15;

(5) that, finally, summing up our conclusions, *the pentatonic scale F G A C D is capable of being "understood" by the audience in many different* (but not all equally probable, by any means!) *ways although the player has nothing but five fixed tones to give.*

The importance of the statement which has been emphasized in the last sentence can be fully valued only by him whose studies in the history of musical theory are far-reaching enough to have made him acquainted with the dogmatic assertions, claiming mutually to exclude each other, of innumerable

writers of the past to the contrary. Not even in the *momentary experience of an individual hearer* do "contradictory" interpretations necessarily exclude each other. Actual (biological) life is *not necessarily logical.* In other affairs, too, contradictory experiences happen to a human being. Does not even the judge in the courtroom sometimes feel that both litigant parties are right? Never mind. Facts are facts even when they do not suit the logician in his arm chair. And so we restate that *pentatonic* scales deserve the epithet "five-tone" and nevertheless possess a *multitude* of tones far exceeding five.

A fictitious story of the invention of a pentatonic instrument

Why, now, did anyone in the history of mankind ever build an instrument having a pentatonic scale? The *five* fingers of the hand may easily have played a suggestive role. But the mere five-ness of the instrument interests us less than *the intervals* which result from the "psychology" of the inventor of this instrument.

In very primitive conditions of civilization the musical instruments first invented must have been *the pipe, the horn and the string* because the materials for these instruments offer themselves readily. (The human vocal organs are not under consideration because they have no fixed sounds.) Hollow plant stems (bamboo) are plentiful in most regions of the earth. The pipe results either as a row of pipes, a Pan pipe (named after a mythological god) or a pipe with finger holes. The Pan pipe is still much used by the country folk in Spain, but also elsewhere. It is composed of several tubes of varying length, tied together and quickly moved sidewise before the player's mouth. The invention of finger holes does away with the plurality of tubes. Horns are a by-product of the industry of the herder; and strings, that is, twisted and dried guts of small animals, sheep, goats, or calves, offer themselves likewise. There are archaeological indications leading to the conviction that the pentatonic scale was especially popular among peoples where pipes (fifes) were popular instruments, for example, in ancient Scotland and in prehistoric Asia Minor, and not among peoples where string instruments were in vogue, as in ancient Greece.

Why, now, did an ancient "piper" invent the pentatonic scale? Of course, an answer to this question can be given only by writing a bit of fiction, a kind of short historical novel, or, rather, prehistorical novel. But as anthropologists we engage in such fiction writing nevertheless, because by the presentation of a concrete and possible case we clear up what otherwise may seem utterly mysterious. Think of the story of a woman who by the application of mud made her basket fit to carry water and then later left it too close to the fire. As if pottery could not have been invented otherwise, too! The pentatonic scale was invented in many ways also, without doubt. Here is one which seems especially plausible.

Imagine a piper who in imitation of his hand tied together five tubes,

made of bamboo or similar stock, all differing by a bit in length. Imagine that, not quite liking the tone relations found, he would attempt to tune these pipes; that is, to cut them longer or shorter until the tone relations *would somehow appeal to his ear*.

Let us call the original five pipes p_1, p_2, p_3, p_4, and p_5, diminishing in length fairly regularly from p_1, which symbolizes the longest, to p_5, the shortest. These symbols play here the same role as that played by key symbols on the piano. We shall see what tone symbols (i. e., numerical symbols) will correspond as the result of his playful inventiveness. Imagine that unwittingly he chose the lowest tone to be of 480 vibrations. He would then discover that, by bringing his lips closer together and blowing a little harder, he can produce the octave, of 960 vibrations, so that he would not tune any other pipes for the Octave (12S). Next to this tone relation the so-called Fifth (7S) would arouse his interest,—formerly mentioned by us as being used for "a kind of unison" by musically gifted savages. He therefore would tune p_4 as 720.

Remember that the only "kinds of unison" (taking this term in the formerly mentioned sense) he has in mind are the Octave and the Fifth. In trying then to have a tone which p_1 (that is, now, 960) could serve as the Fifth, he would tune p_3 as 640. In further trying to have a tone which is the Fifth to p_4, he would tune p_2 (by "overblowing") as 1080 vibrations, but would also obtain in this way the tone of 540 vibrations in the same pipe p_2. The last pipe, p_5, he then discovers, can be tuned as the Fifth of p_2 (now 540); and thus he tunes p_5 as 810, that is, three times 270.

It is a rather remarkable fact, fairly well established by history, that the earliest peoples among whom pipes were popular musical instruments, the Chinese, the Western Asiatics, the earliest Greeks in the "Pythagorean" period, tuned their pipes in this way. The following table will bring the result more clearly before our eyes.

	p_1	p_2	p_3	p_4	p_5
I	480	540	640	720	810
	960	1080			
II	48	54	64	72	81
III	3	27	1	9	81
IV	C	D	F	G	A

Of course, the pipe which in line IV is called A would still be shortest, and the pipe C still the longest. The five piano keys C D F G A are quite arbitrarily given to make the reader think of something concrete and familiar which produces a similar group of tones.

In the same way as this imaginary savage a Scottish piper may have become accustomed to the "pentatonic scale. He transferred it then to a *single pipe* having four holes bored which, in addition to the end hole, enable the piper to produce the same tones as if he had five tubes.

The above table gives in line I the actual vibration rates which we imagined, and in line II the same reduced to one-tenth. In line III we find the usual symbols, resulting from the elimination of the factors represented by pure powers of 2.

The drone bass is perhaps the earliest experience of mankind with harmony

Problem 28: The Scottish piper invents the drone bass and discovers experimentally that C in a lower octave makes a very acceptable drone bass to the tones indicated in line IV of the table. After some experience with the music in the pentatonic scale accompanied by this drone bass, he perhaps decides that he perfers a *slightly* (only by a few cents) altered tuning of one of the four tones D, F, G, or A. Question: which of them does he perhaps alter and how? That is, *what* tone symbol must we now substitute for *what?* And why? Answer:

It is clear, of course, that the Scottish piper, although he commands only five tube lengths, (on the piano one should say five strings or keys), is thereby restricted only to five symbols in the scale of keys or tubes, not however to five physical tones. Opening the lips more and blowing with considerably less pressure may produce a lower octave. This is an advantage of the fife over the piano. The piper can therefore without difficulty play Auld Lang Syne on his instrument.

Another primitive scale

Let us amuse (and instruct) ourselves now with another bit of archaeological fiction. Imagine a herder who has for one reason or another cut off the tip of an unusually long horn of one of his slaughtered animals and who has discovered that by pressing the lips tightly against the opening and blowing forcefully, one can produce tones. Acoustics teaches us that all vibrating bodies vibrate simultaneously in divisions and not merely as individual wholes. The "thinner" (using the word in the broadest popular sense) they are, the more readily they divide into more and more numerous vibrating divisions. Air is very "thin", molecularly (and, when in a tubular container, even geometrically). And it is the air which is the vibrating body in the horn, not the horn itself. Every "sound" is therefore a compound, some sounds more conspicuously, some less so.

Modern acoustics says that such a compound consists of "partial" tones. (The more old-fashioned and rather "barbarous" and illogical term "overtones" is avoided in this book's terminology.) The partial tone corresponding to vibration of the whole is called the first, or simply "one." The tone corresponding to the halves of the body is called the second, or simply "two". When we speak here of these partial tones "one, two, three," and so forth, this has no reference whatsoever to the number symbols used in a real musical sense, and it is therefore advisable to write these partial tones in Roman numerals. I, II, III etc. are therefore the partial tones of one and the same vibrating body

and have no directly musical meaning, but directly only a mechanical meaning.

The herder soon discovers that the lowest partial tones I and II are *heard* once in a while, but that it is hard to *produce* them on the horn. He also discovers that such higher partial tones as VIII, IX, X, XI, etc. are increasingly hard to produce. (A modern physics professor could tell him the mechanical reasons for that.) But there are some half-dozen or so of tones whose production is not unduly hard.

The herder's selection of a "scale" is thus necessarily limited on both ends of the series by the increasing difficulty of producing the tones. But his selection of a "scale" is also limited by the advantage of having the "octave" interval at his disposal, or the disadvantage of losing the octave interval by not practicing a suitable series of partial tones. The longer the tone series, the more practice is of course necessary in order to get the "knack" of producing varying tones at will.

Probably, then, the herder tries the series III, IV, V, VI, which contains the octave. He also tries the series IV, V, VI, VII, VIII, which also contains the octave. He also tries the series V, VI, VII, VIII, IX, X, which also contains the octave. The further series from VI to XII he is likely to discard very soon as becoming too difficult on account of XI and XII. The third mentioned series would be easy enough on a horn that is very uniform in width, but less so on a horn that tapers, the range up to X being rather difficult. Although he has here the charm of so great a number as six tones, he gets tired of obtaining the needed skill and tries the series IV, V, VI, VII, VIII, which is easier, but not too easy, by any means. He also tries III, IV, V, VI and finds this still easier. And thus, he selects the scale III, IV, V, VI.

This herder then has become the first "bugler" of mankind. Up to our own times the most popular bugle tunes (used for military and other signals, especially) have been in his scale. What is this herder's scale?

The actual vibration rates of the four tones selected by the herder are 3, 4, 5, 6 (this whole series multiplied by any suitable common factor if you want the rates per second, per minute, or per any other *particular* time unit). You now divide each member of the series by 2 as often as possible, in order to get the tone *symbols*. This gives "3, 1, 5, 3" as the tone symbols we want; or rather "3, 1, 5," since we do not have to repeat symbols. The bugler's (present) scale (of prehistoric origin) is therefore "3—1—5." But remember, just as pottery has many origins, so have bugle calls. This origin is only one,—one that easily makes a story like the one just told.

Problem 29: Find among A, B, C, D, E, F, G as many "triplets" of key-symbols as possible, each "triplet" being capable of representing the tone-symbols of *this bugle scale* "3—1—5."

A third kind of primitive scale

We recall again our former statement that the pipe, the horn, and the

string are the instruments most easily made from material available under primitive conditions. Let us then tell as a third piece of archaeological fiction the story of the string. Among the prehistoric Greeks the tradition had developed that a string instrument should preferably have four strings tuned so that certain tone relations were obtained. We happen to know from what is mentioned by Greek writers about "the" ancient string instrument, that (most commonly) its four strings were tuned so that the three successive intervals, from the lowest to the highest tone, were the following:

— S — 2S — 2S —

The dashes above represent the four strings, or their four tones. The symbols between the dashes give the approximate size of the intervals in units of 100 cents. Why these and not other intervals? Let us invent a plausible story to answer the question.

A herder happens to notice that a cord, made of a twisted and dried calf gut and used by him for the same multiple purposes for which we use a rope in our domestic life, is moist and in need of being dried. He runs it several times over a board which happens to be warped, so that the cord does not touch the slightly concave surface of the board.

Before continuing the story we might explain to the reader why we chose a calf gut rather than a sheep or goat or cat gut. Merely because it gives us a chance to mention the etymology of the word "violin", although this is not of Greek, but of Latin origin. An animal which is sacrificed to the gods is a victim, of course. If the animal sacrificed was small, it was called in Rome by the diminutive form "vitula." Since these animals were usually calves, the word "vitula" came to mean calf. Later a string instrument for which a calf gut was used, was referred to by the same word; but the *t* dropped out and the *u* changed into *o*. Thus the names "viola" and "violin" came into existence.

We return to our story. The herder, in order to test his cords for being dry enough, plucks them and discovers that they give different sounds. He discovers that it is possible to control the sound of the gut by varying the tension. But there are limits to this procedure. First, if the tension is made too great, the gut breaks. Second, if the tension is made too weak, the tones become too weak, too dull, even cease completely. So the usable sounds are within *a limited range*. And we understand at once why the herder cannot use the octave; that range is too great. We understand also why the ratio of the tone symbols "1" and "3" appears as "3↗ 1" and not as "1↗3". The former has *the smaller range*, of 5 S as against the 7 S of the interval "1↗3".

Problem 30: The herder of the above story attempts, after tuning the lowest and the highest string in the ratio 3:4 (symbol 3↗1), to tune an intermediate string so that it represents, either with the lowest or with the highest (double problem!) string, the tonal relation of the symbol "1-5". Of course, this can mean only 1↗5, for 5↗1 is an interval of 8 S which cannot fall within

the "limit" of 5 S represented by 3↗1. Tell how far the new and intermediate tone is distant in S units from the highest or lowest. The double problem of course has a double solution. Letting the dashes stand for the tones, indicate the intervals by writing in S or 2S or 3S, etc., and write the three tone symbols over the three dashes:

First answer: _____ _____ _____

Second answer: _____ _____ _____

We continue our story. We remember that the smaller the numbers, the more likely the musician is to make use of the corresponding tones. The numbers 1-5 are smaller than the numbers 3-5. For some reason, by accident or otherwise, the herder has at one moment succeeded in tuning his three strings so that they represent the scale 15, 1, 5, in accordance with *one* of the solutions of problem 30. He attempts to tune a fourth string so that it represents with 15 or with 1 or with 5 of the above scale the tonal relation "3 - 5". Of course, 3 - 5 can here mean nothing but 5↗3, for 3↗5 would far exceed his total range of 5 S, to which we have already found him committed. He tries the three following tetratonic scales, listening to them:

I	15	—	1	—	9	— 5
II	75	—	5	—	3	— 25
III	45	—	3	—	25	— 15

Tetratonic scales generate the historical scale of the double tetrachord

Of course, he knows nothing of the above numbers. He merely listens to the corresponding tones. But we, who know these numbers and what they signify, will not be at all astonished to learn from his occasional remarks that the herder is less pleased with the scales II and III (whose symbols are larger numbers) than with scale I, "15, 1, 9, 5".

The last mentioned scale (15, 1, 9, 5) is that of the most common of the ancient "tetrachords", that is, the *four-string* instruments of the Greeks. The insight thus obtained is indispensable if we wish to understand the origin of the common musical theory and practice taught in all the music schools of the Western Civilized World of today, as we shall see.

Let the reader start again from the solution of problem 30 and prove what would have happened in the story just told if our inquisitive musical savage had succeeded in tuning his three strings, not 15↗1↗5, but in accordance with the other solution of problem 30, and had then added a fourth string for the purpose of realizing the tonal relation 5↗3.

Answer:

It now happens in our story that the herder visits a brother and tells him of his invention of a string instrument. The brother looks around for material to make a similar instrument. He finds a board and a chord, but the latter is a twisted gut of a smaller animal, say, a cat. The result is that when

the two brothers have finished tuning the new "tetrachord" in the (now traditional) scale "15, 1, 9, 5", the tones are much higher than those of the former "tetrachord", because the cord is of smaller size. The smaller the cord which is tied over a board of a given length, the higher it must be tuned to give out the strongest possible sound, for mechanical reasons. The brothers convince themselves of this fact when they take the new instrument to the first brother's tent in order to compare it with the first instrument. The compass of the first instrument lies lower.

In playing with the two instruments it occurs to the brothers that they might combine the two into one. They get this notion mainly from discovering that (accidentally) the lowest tone of the higher instrument does not sound different from the highest tone of the lower instrument.

Problem 31: What seven-tone scale, expressed in numerical tone symbols, results from combining two scales "15, 1, 9, 5" by means of identifying the tone "15" of one scale with the tone "5" of the other scale? Answer:

The origin of our white key piano scale

The musical brothers in our story now have a seven-tone scale based on the common Greek tetrachord. When they play this "harp", as we may name the larger instrument, they become aware of being almost (the span measures now 10S) able to give reality to that so conspicuously important interval, the octave. All that is necessary is to find an unusually thin or an unusually heavy chord and then to add a tone either 2S higher than the highest of the seven or one 2S lower than the lowest of the seven. They try both methods and find the total scale of a more pleasing character if they obtain the octave by adding the lower tone. As we watch them doing this, we look at the answer of problem 31. We understand now their choice. They have used *twice* the tone expressed by the *smaller number symbol*, not the tone symbolized by the larger number. The reader understands, of course, that our whole endeavor is directed toward deriving the history of music from the "preference", so to speak, for smaller number symbols, other conditions permitting a choice.

It now occurs to the brothers to give to the eight strings (and tones) of their complete harp successively the letters of the alphabet as names. But for the reason which we have repeatedly mentioned they give to the eighth string the name "A". This finishes our story of the invention of the scale which fortuitously was called by the Greeks, and has continued for two thousand years to be called, *the diatonic scale*, i. e., the "thoroughgoing" scale.

For the performer the white key piano scale contains within it innumerable smaller scales (when he leaves certain keys untouched) and larger scales (although he touches with his fingers only seven keys of the instrument)

We remember that on a previous page we pointed out that the pentatonic scale is contained in the diatonic scale, indeed contained therein twice. We

have also learned that the bugle scale is contained in the diatonic three times. This should make it clear why modern musicians have confined themselves to speaking of *the diatonic scale* without mentioning, or rarely mentioning, the other two. The fifer's scale and the bugler's scale are *archaeologically* as important as the harper's scale, but *historically* for the European-American civilization the harper's scale, based on the two Greek tetrachords, is *sufficient* for understanding the present practice of our musicians.

Problem 32: Write in one line the first seven letters of the alphabet, separated by long dashes. Write, in the line *above*, the seven symbols of problem 31 so that each letter (key-symbol) has a tone symbol directly above it.

Identify the tone-symbol 5 with the key-symbol A. Then write in a third line, below the dashes of the second line, the size of the interval in cents, not measuring the intervals of the key-symbols in tempered tuning, which would result in mere multiples of S, but measuring the interval from each tone-symbol to the next tone-symbol (of the first line) by the aid of our table of spans, in cents. See that each dash has the proper number of cents standing directly below it. Answer to suit problem 31 only:

5							5
A —	B —	C —	D —	E —	F —	G —	A
A	B	C	D	E	F	G	A

When we discussed in this "musician's arithmetic" the pentatonic scale, did we not point out that it was "pentatonic", that is, a *five-tone* scale, *only* with respect to *key-symbols?* Only in the practical sense of the musical *performer* was this scale a five-tone scale. In the psychological sense, that is, for the audience, we saw that the scale called pentatonic could as well be a six-tone or seven-tone or eight-tone (and so forth) scale, although *penta* means five, and to the performer decidedly so. The same is true for the diatonic scale of seven key-symbols. This scale, without actual additions by the performer, can easily be interpreted by the audience otherwise than above, say, as an eight-tone, nine-tone (and so forth) scale.

The fact that the diatonic scale has become, say, a fifteen-tone scale played on only eight strings (without the octave string only seven) does not, however, force the composer and the audience to perceive it constantly and invariably as composed of all these fifteen tones. In a particular melody some of these fifteen tones may not make their appearance. That is, some of the fifteen, maybe eight of them, may remain unused by the composer and the audience. And those seven tones still used may not form even a seven-tone scale in the ordinary sense of the word: they may include during *the performance of a particular piece* only a part of the seven strings making up that *particular* diatonic scale of the inventive brothers of our archaeological fiction.

The word "particular" in this reference to the diatonic scale is justified by the fact that the general scale A B C D E F G is, as we have made clear,

a scale of many *different meanings*. A few problems to be solved will help us to get a more complete notion of this multiplicity of meanings.

The enormous multiplicity of possible interpretations of any merely performer's scale

We might for this purpose solve the following problem. What are *the seven smallest* tone-symbols by means of which the key-symbols A B C D E F G may be represented? The word "smallest" in this question, referring *to a group and not to one* thing, must be defined. Let us use it *here* in this drill problem in the sense of "giving the smallest total when the seven particular numbers are added together". In *that* sense, if for example we were dealing with three only instead of seven numbers, the group 3—13—15 would be one of smaller numbers than the group 9—11—13, for *by definition* the former three are added to give only 31 whereas the latter three, added together, give more than 31.

In order to solve this problem it is well to use the following trick.[4] Write down all the *odd* numbers from 1 up which are composites of (the powers of) nothing but 1, 3, 5, and 7. This series of numbers (tone-symbols) starts as follows: 1, 3, 5, 7, 9, 15, 21, 25, 27, 35, 45, 63, 75, 81, 105, 125, 135, 175, 189, 225, 243, etc., etc. (The reason why, stopping at 7, we did not let *the factor* 11 enter into the composites is *not a dogmatic one*, but simply one of practical convenience as explained on previous pages.) Now distribute this series of numbers over twelve successive key-symbols as shown below, but (further) omit those numbers which, when computed with reference to either 1 or 3, in your opinion fall more nearly *between* two key-symbols than near to any one of them.[5] If the following table does not agree with your opinion as to what might be conveniently omitted, you are free to add to the table as many as you wish, for this is only a drill problem:

	81 675 729 189 405 225 243 63											
	5	21	45	3	25	27	7	15	1	135	9	75
III, IV, etc.	A	Ais	B	C	Cis	D	Dis	E	F	Fis	G	Gis
I	E	F		G		A		B	C		D	
II	A	Ais		C		D		E	F		G	

Now write on a slip of paper of the same dimensions either the key-symbols of the diatonic scale C D E F G A B C (I above) or our *key-symbols* of the

4 First described in The *Psychological Review*, VII(1900), p. 251. Later in *University of Missouri Studies*, Original Series I, No. 1 (1901).

5 Those omitted (but *not dogmatically* omitted) are 35, 105, 175, 315, etc., all multiples of 35, and perhaps a few other numbers.

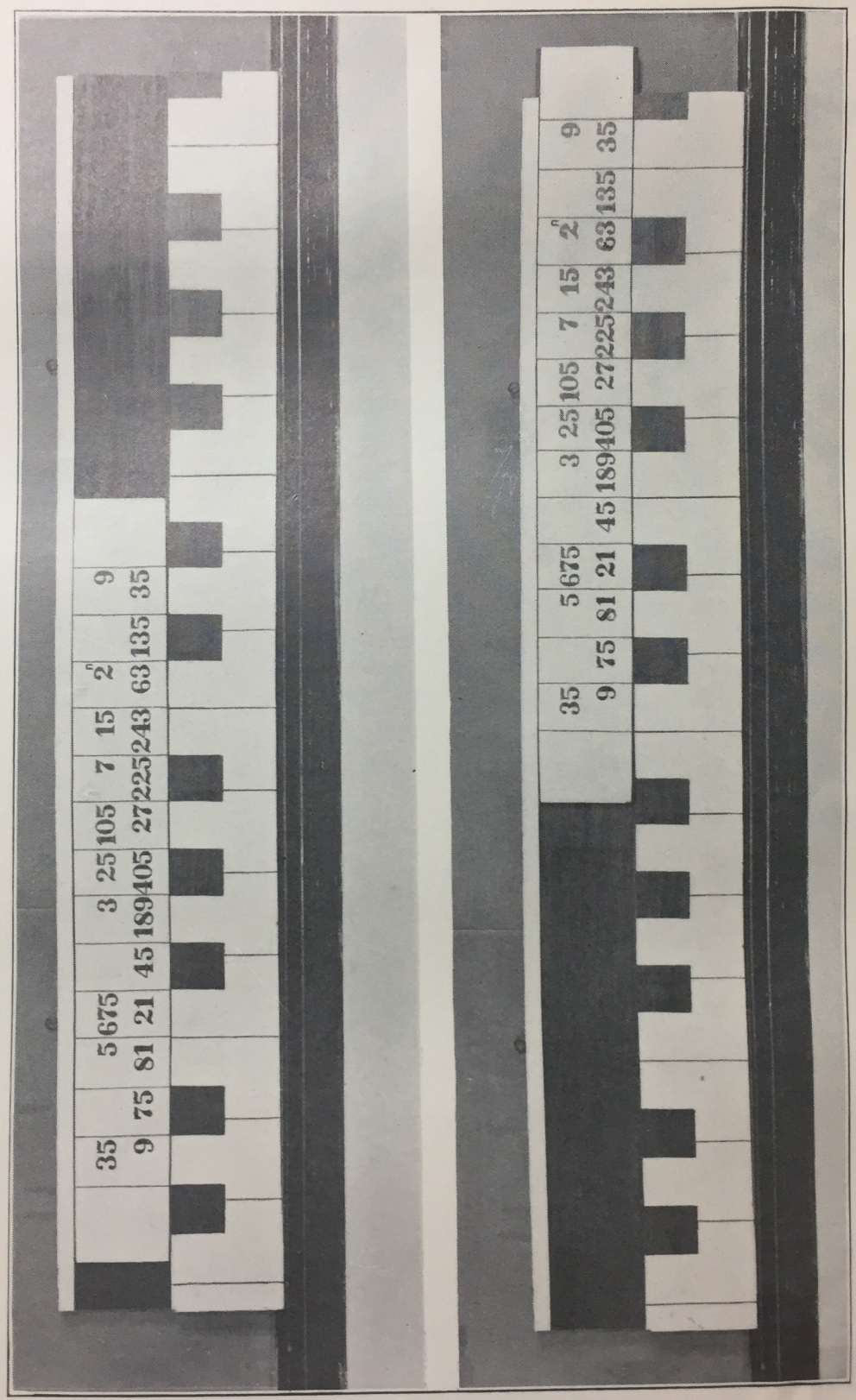
diatonic scale in F major (II above) or *any other* (tempered) diatonic scale, preferably extending over a range of two octaves (III, IV, etc., to be added if you wish). Put the slip under the tone-symbols, that is, at the level of I or II or III, and slide it to the right or left. When you add up the numbers indicated in *this* or *that* or *another* position of the sliding slip, you will convince yourself that you cannot solve the problem of the minimum sum of the tone symbols better than by sliding your diatonic scale in the position shown to the right of I or II.[6] In no other position do you get smaller numbers. And the solution of the drill problem above, proposed to lead to a comprehension of the multiplicity of meanings of the "diatonic scale" by presenting to us *one* of those many meanings, is *in this case* the following:

1	9	5	21	3	27	15	1
C	D	E	F	G	A	B	C

The total of all the seven numbers is in this selected meaning of the diatonic scale only eighty-one. In every other placement of the slide you get a larger total. Try to prove this statement is wrong if you can. This then is *that* diatonic scale which can claim the distinction of being made up of the smallest numbers.[7] But do not exaggerate the importance of this distinction. Other diatonic scales (in the sense of being made up of other numbers) have their own and other claims, as we shall gradually understand better and better.

6 The choice of "Ais", unexpected by musicians, rather than "B^b" in the case of II is intentional *just because* it runs counter to the "scale habits" of the musical kindergarten. We desire to emphasize that these are piano key names only, not theoretical symbols.

7 Compare H. W. Poole in *Silliman's American Journal of Arts and Sciences* IX (1850), pp. 68-83. Poole seems to be the first who ever mentioned this scale.



A SLIDING DEVICE FOR CLASS ROOM USE IN THE UNIVERSITY OF MISSOURI

Which is the best drone bass of the last mentioned scale?

(Sample) Problem 33: Take a musical instrument and run down that last scale of seven tone-symbols from B to C (or up from C to B) and accompany this "tune" by a drone bass in the Scotch fashion. If you should choose for this drone the symbol 5, how successful to the ear would that be? In order to get a basis of theoretical comparison, do this: Add together the symbols of the two tones of each pair, the drone being one term of each pair. Then add together these seven numbers to find the total. Answer: First pair 5 plus 1, that is, 6. Second pair 5 plus 9, that is, 14. Third pair 1 plus 1, that is, 2. (It must be noticed, that the third pair, 5 and 5, *must have its terms reduced* before they are added.) Fourth pair 5 plus 21, that is, 26. Fifth pair 5 plus 3, that is, 8. Sixth pair 5 plus 27, that is, 32. Seventh pair 1 plus 3 (the ratio 5:15 is reduced to 1:3), that is, 4. Now add together 6, 14, 2, 26, 8, 32 and 4. The result is 92, which is to be compared with the result in other cases.

Problem 34: Use a drone as in the preceding problem, but choose for it 3. What is the sum of the seven sums? Answer:

Problem 35: Use a drone as in the preceding problems, but choose for it the tone-symbol 1. What is the sum of the seven sums? Answer:

If you should find in the history of music that someone, in order to accompany the diatonic scale CDEFGAB, had decided to use the particular tone of G as a drone bass, would this become more plausible to you after comparing the results of problem 33, 34 and 35?

How could Rameau's illogical doctrine that "there are only two triads" impose so long upon the musical world?

(Sample) Problem 36: Find all the possible "duads" (combinations of two each) which can be formed of the symbols 1, 3, 5 and 7. Answer: (a) 1 — 3, (b) 1 — 5, (c) 1 — 7, (d) 3 — 5, (e) 3 — 7, (f) 5 — 7.

Problem 37: Find all the possible "triads" (combinations of three each) which can be formed of the symbols 1, 3, 5, and 7 by "combining every one of the preceding duads" with every other one, to wit: ab, ac, ad, ae, af, bc, bd, be, bf, cd, ce, cf, de, df, ef. Proceed as follows: identify by using a common name (so-called denominator) the first symbol of a with the first of b, the first of a with the second of b, the second of a with the first of b, the second of a with the second of b, as indicated by the Roman numerals of the four columns. A few of the answers are given below; continue, giving the remainder and filling in the spaces left for them.

	I—I	I—II	II—I	II—II
ab	1 — 3 — 5	1 — 5 — 15	1 — 3 — 15	3 — 5 — 15
ac	1 — 3 — 7	1 — 7 — 21	1 — 3 — 21	3 — 7 — 21
ad	3 — 5 — 9	3 — 5 — 15	1 — 3 — 5	5 — 9 — 15
ae				
af	5 — 7 — 15	5 — 7 — 21	5 — 15 — 21	7 — 15 — 21
bc				
bd	3 — 5 — 15	3 — 5 — 25	3 — 15 — 25	1 — 3 — 5
be				
bf				
cd				
ce	3 — 7 — 21	3 — 7 — 49	3 — 21 — 49	1 — 3 — 7
cf				
de				
df				
ef	15 — 21 — 35	15 — 21 — 49	15 — 35 — 49	3 — 5 — 7

We have above sixty cases; but some of them appear more than once. So that it is only for convenient reference when we speak below of "sixty".

Problem 38: Regard each of the above "sixty" triads as a "scale". Imagine that you want to write in each of these scales "two-part-harmonies". This gives for each of these triad scales three possible harmonic combinations of two tones, or duads. For example, in the fifty-eighth, 15 — 21 — 49, these are the three duad harmonies: 15 — 21, 21 — 49, 15 — 49. Now treat these as you treated the pairs of problem 33, add the ratio terms of each pair together and then add the three sums together. For example, 21 & 15 gives 12 (that is, reduced, 7 plus 5), 21 & 49 gives 10 (that is, 3 plus 7) and 15 & 49 gives 64 (that is, 15 plus 49). And the total of 12, 10, and 64 is 86. Do the same with the other fifty-nine triad scales. Answer:

If you should find in the history of music that some one (to wit, Rameau) had asserted that the two triads 1 — 3 — 5 and 3 — 5 — 15 alone were "legitimate" and that all of the others of the table of "sixty" were worthless mongrels that should be given a bad name (such as the name of "dissonances"), you of course would not agree with him. But you might convince yourself that he was at fault only in exaggerating, that he would have been perfectly right in pointing out that "something" places these two triads 1 — 3 — 5 and 3 — 5 — 15 before the others and that all the others possess that something in a lesser degree only. Prove it by comparing the "sixty" results of problem 38 and thus somewhat exonerate Rameau. What is it? Of course, the ultimate answer lies in the chemistry of the nervous system!

One of the various factors possible, which determines the key-note of this scale

In reading above that particular diatonic scale which just precedes prob-

lem 33, were you not astonished to see that the first letter written there by the author, arbitrarily, was no longer A, the first letter of the alphabet, but C? The present problem will not yet tell you the full reason why that was done, but will lead in that direction.

Problem 39: You concluded—did you not?—from the results of problems 33, 34 and 35 that the tone "3" would be the most suitable drone bass for the scale 1, 9, 5, 21, 3, 27, 15; that is, that G would be the most suitable drone bass if this series is called C, D, E, F, G, A, B. Reverse now this statement, remembering that the scale A B C D E F G has "many meanings" in tone-symbols. The reverse statement becomes then the answer to the following question. If G is used as a drone bass, what tone-symbols are most likely to be "the meaning" given by the audience to the key-symbols A B C D E F G? Answer:

Problem 40: The experience of musical experts in general and also experiments made on numerous subjects by Meyer and independently by Farnsworth (compare Appendix 10) have proved that when two tones are presented to the ear there is a remarkable tendency of the audience—other influences being balanced—to prefer an ending on the tone represented by a smaller number as its tone-symbol. Accepting this statement, do you see the reason why a diatonic scale *interpreted like the one* just before problem 33 should be written so that a special importance of the key-symbol C be indicated by placing C at the head? What is the reason? Answer:

In agreement with problems 39 and 40 it is an easily observable fact that the diatonic scale A B C D E F G when forced by the drone bass G into the interpretation 27, 15, 1, 9, 5, 21, 3, has as its preferred ending an ending on C. Everybody would be willing in such a case to call C the key-note of this scale, because of this ending-preference which gives it a special significance and makes it the chief note of the scale. (Key-note means nothing but chief note.) The ending-preference is only one significance which may be had by the chief note in a scale, for besides the fact that a note makes a good ending it may be significant in many other respects. Indeed, one can give a certain note a special *significance* in a tune quite artificially, for example, by introducing it arbitrarily with unusual frequency. It would then be a rather *artificial* key-note, but it would be a *key-note* just the same.

Another and less simple one of the various factors which may determine the key-note of a scale

Problem 41: What would be in the very simple scale 1 — 3 — 5 the *most naturally* chosen key-note, in accordance with the statements made in problem 40? Answer:

Problem 42: Why would in the very simple scale 3 — 5 — 15 the tone "15" be the worst chosen key-note? Answer:

The experiments made by Meyer and by Farnsworth (compare Appendix 10) have proved, however, that the ending-preference between two tones is a *very conspicuous* effect only when two conditions are both fulfilled: when one of the tone-symbols is a pure power of 2 and the other tone-symbol is a rather small number (which is not a pure power of 2). Since we express the pure power of 2 by the symbol "1", the same fact may be restated thus: the ending-preference is a *very conspicuous* experience *only* in tone pairs such as 1 — 3, 1 — 5, 1 — 7, 1 — 9, 1 — 15, and is a much less conspicuous occurrence in others, such as 3 — 5, etc. And, again, it is among the five pairs just mentioned *the more conspicuous, the smaller the number* symbolizing the term *which is not* a power of 2. Let us make an application of these rules.

Problem 43: In the very simple scale 3 — 5 — 15, for *what various* reasons is 5 the best chosen key-note? (Remember that, in pairing, 3 — 15 reduces to 1 — 5 and 5 — 15 reduces to 1 — 3.) Answer:

We have already mentioned in connection with problem 38 that Rameau, before publishing in 1722 his famous book "Traité de l'Harmonie", had become impressed with the two little scales 1 — 3 — 5 and 3 — 5 — 15. He had also become impressed with the fact that the best chosen *key-note* is 1 for 1 — 3 — 5, and 5 the best for 3 — 5 — 15.[8] He now discovered that *these two* scales could be fitted (accidentally) into the traditional Greek diatonic scale in a rather *mysteriously symmetrical* manner.

Since the people of his century, as well as he personally, were strongly inclined toward mysteries, he decided that he had discovered a great "secret of nature," and he built upon it a theory of music which we must now struggle hard to rid ourselves of. It has established itself firmly in the grammar schools of musicians. It has become a sort of second nature with them. Let us give this *almost thrilling* (don't take that seriously!) secret of nature discovered by the mystic Rameau, a name which is as stale and unemotional as possible. That will help to prevent us from drawing logically unjustifiable conclusions from it. Let us call it the "theorem of Rameau" and make ourselves familiar with it in the next problems.

Problem 44: How many times, and by what key-symbols within the diatonic scale A B C D E F G, can the little scale 1 — 3 — 5 be represented, and which are the key-symbols representing in each instance "the key-note" of the little scale 1 — 3 — 5? Answer: (a) — — times, (b) by the key-symbols — — — — — — — —, (c) with the key notes — — — —.

Problem 45: How many times, and by what key-symbols within the diatonic scale A B C D E F G, can the little scale 3 — 5 — 15 be represented, and which are here the key-symbols representing in each instance "the key-

8 Unfortunately, when musicians talk of such *little* scales, they do not call the *chief* note "the key-note", but "the fundamental", although the latter term is meaningless.

note" of the little scale 3 — 5 — 15? Answer: (a) — — — times, (b) by the key-symbols — — — — —, (c) with the key-notes — — — — —.

Problem 46: What tone-symbols must represent the diatonic scale A B C D E F G in order to have this scale contain within itself all the scales 1 — 3 — 5 found in the answer to problem 44? Answer:

Problem 47: What tone-symbols must represent the diatonic scale A B C D E F G in order to have this scale contain within itself all the scales 3 — 5 — 15 found in the answer to problem 45? Answer:

Problem 48: What tone-symbols must represent the diatonic scale A B C D E F G in order to have this scale fulfill both the requirements of both the problems 46 and 47? Answer:

After having solved problem 48 (but not before that) you will profit from reversing problem 44 in the following manner.

(Sample) Problem 49: A combination of *three little scales* 1 — 3 — 5 in such a way that all three be performable in the diatonic scale, is possible only by combining the three so that the first 1 — 3 — 5 equals F — C — A (3—9—15), the second equals C — G — E (9 — 27 — 45), and the third equals G — B — D (27—81—135). Imagine now—in figurative language—that these three little scales are a political committee of three members and have to start their committee business by electing by secret ballot one among them to be chairman. Providing that no one votes for himself, which of the three is most likely to receive two votes (of the two others) if balloting continues until one gets two votes?

Solution: It is necessary to investigate the relations existing between the first and the second committee-man, between the second and the third, and between the third and the first. This may be done by submitting each of these three pairs to the same scrutiny which we have exercised in problem 33. That is, we take F — C — A, the first "committee-man" (scale of three tones), and C — G — E, the second, and form all the nine pairs of each constituent of the first (3, 9, and 15) and each constituent of the second (9, 27, and 45), proceeding exactly as in problem 33.

First pair	1	plus	3	(reduced from	3 : 9),	that is, 4
Second pair	1	plus	9	(reduced from	3 : 27),	that is, 10
Third pair	1	plus	15	(reduced from	3 : 45),	that is, 16
Fourth pair	1	plus	1	(reduced from	9 : 9),	that is, 2
Fifth pair	1	plus	3	(reduced from	9 : 27),	that is, 4
Sixth pair	1	plus	5	(reduced from	9 : 45),	that is, 6
Seventh pair	5	plus	3	(reduced from	15 : 9),	that is, 8
Eighth pair	5	plus	9	(reduced from	15 : 27),	that is, 14
Ninth pair	1	plus	3	(reduced from	15 : 45),	that is, 4

The sum of the nine values on the right side of the above tabular state-

ment is 68. This sum, 68, we shall tentatively regard as a comparative measure of the "unity", the friendship, so to speak, existing between the scale 3 — 9 — 15 (F — C — A) and the scale 9 — 27 — 45 (C — G — E).

Continuation of the solution of problem 49: The student will now take C — G — E, the second "committee-man" (scale of three tones), and G — D — B, the third, and in a table similar to the above form all the nine pairs of each constituent of the second (9, 27, and 45) and each constituent of the third (27, 81, and 135).

First pair	(reduced from 9 : 27), that is,
Second pair	(reduced from 9 : 81), that is,
Third pair	(reduced from 9 : 135), that is,
Fourth pair	(reduced from 27 : 27), that is,
Fifth pair	(reduced from 27 : 81), that is,
Sixth pair	(reduced from 27 : 135), that is,
Seventh pair	(reduced from 45 : 27), that is,
Eighth pair	(reduced from 45 : 81), that is,
Ninth pair	(reduced from 45 : 135), that is,

What is now the sum of the nine values on the right? Answer:

Further continuation of the solution of problem 49: The student will now form all the nine pairs of each constituent of the third little scale G — D — B (27, 81, and 135) and each constituent of the first, F — C — A (3, 9, and 15).

First pair
Second pair
Third pair
Fourth pair
Fifth pair
Sixth pair
Seventh pair
Eighth pair
Ninth pair

What is *now* the sum of the nine values on the right, and how do the sums obtained from the *three* tables compare? Answer:

If we could now have a spiritualistic sitting, call the ghost of Rameau, and ask him which of the three little scales F — C — A, C — G — E, and G — D — B ought to be the chairman of the committee of three, he would unhesitatingly tell us that the second, C — G — E, ought to be the chairman. Without asking him the question, we know from his books what he would answer, for he calls the other two merely dominant chords, but the second one he always treats as the very ruler of the piece of music. "Dominant" of course means in musical terminology something like "captain" in the army or "committee-man" in politics. But the second chord, or second little scale,

as we called it, is comparable to the very general of the army, or the political boss.

Indicate in what manner Rameau's theorem, or opinion, is justified by comparison of the three sums used for measurement and obtained in solving problem 49. If the second committee-man is alone likely to receive the votes of the two others, then the friendships between the second and the first and between the second and the third must be greater than the friendship between the first and the third. Is that true? Do the sums found indicate it?

Compare with the result just obtained the conclusion of the paragraph preceding problem 33.

Problem 50: In accordance with problem 41 the key-notes of the three little scales F — C — A, C — G — E, and G — D — B would most naturally be chosen as what? And if of these three key-notes *the one belonging to the chief* scale (to the "chairman" of that committee) were chosen as "the" key-note of the whole diatonic scale A B C D E F G, which of these key-symbols would be the key-note of the scale? Answer:

Problem 51: A combination of *three little scales* 3 — 5 — 15 in such a way that all three be performable in the diatonic scale of problem 48 (look back!) is possible only by combining them so that the first 3 — 5 — 15 equals F — D — A (3 — 5 — 15), the second equals C — A — E (9 — 15 — 45), and the third equals G — E — B (27 — 45 — 135). Imagine now, as in the previous story of a committee, that these three have to elect a chairman. Which one is likely to get two ballots although no one votes for himself? Construct three tables as in solving problem 49 and then give the answer:

On the basis of the last answer, indicate why Rameau in his writings regarded F — D — A and G — E — B as if these little scales were mere captains in the army (merely dominant chords in the music), but C — A — E as the very general of the army (the chief chord of the same music) who has the confidence of all the captains and thereby of the whole army. Is Rameau justified? Do the numbers indicate that?

Problem 52: In accordance with problem 43 the key-notes of the three little scales F — D — A, C — A — E, and G — E — B would most naturally be chosen as what? And if of these three key-notes *the one belonging to the chief* "scale" (to the "chairman" of that committee) were chosen as "the" key-note of the whole diatonic scale A B C D E F G, which of these key-symbols would be the key-note of the scale? Answer:

Recapitulation: Some historically significant ones among the various factors which may determine the key-note of a scale

You noticed (or, if you did not, look back and notice now) that just preceding problem 33 we suddenly deviated from the previous custom of writing A B C D E F G and, instead, wrote the key-symbols in that place as follows: C D E F G A B C. In problem 40 you gave an answer justifying this

apparently arbitrary treatment of the alphabet. The diatonic scale accompanied by *the drone bass* of G could not help assuming a meaning which made the choice of C as the key-note inevitable.

When Rameau, however, gave special emphasis to the tone combination 1 — 3 — 5, simply by letting it be heard very frequently as a group of tones forming *chords* in various positions (1 ↗ 5 ↗ 3, 3 ↗ 1 ↗ 5, and 5 ↗ 3 ↗ 1), or even merely arpeggios, he brought about an effect very similar to that of the drone G in so far as the tone C was given a significance above that of the other six. You saw in problem 50 how there C became the most naturally chosen key-note, by a vote of all the tones, so to speak.

In problem 52, however, you saw how the tone A rather than any of the other six becomes in the music the most naturally chosen key-note. The composer has merely to emphasize that other combination, 3 — 5 — 15, *by the use of various "chords" or "broken chords" of this composition.*

That the choice of the key-note depends on a VARIETY of conditions fulfilled or unfilled in a particular instance must now be clear. In ancient and medieval music *seven* "modes" were distinguished according as in unisonous singing *any one* of the seven notes A B C D E F G was *arbitrarily enforced* as a key-note. We have above (in problems 34, 49 and 51) learned three specially effective means of *naturally*, *structurally*, *not arbitrarily*, enforcing a key-note. First, we enforced C as a key-note, using either of two means; next, we enforced A, by the third means, as the key-note. Let us restate the three means in the order opposite to the one in the last sentence. The use of various chords of the make-up 3 — 5 — 15 makes A a relatively "natural" choice as key-note. On the other hand, the use in the composition of various chords 1 — 3 — 5 makes C a relatively natural choice as key-note. These two methods were discovered by Rameau and make him distinguished in the history of music aside from his just fame as a composer. The use of the drone G again makes C a relatively "natural" choice of a key-note. To the Greeks and medieval musicians none of these three means of naturally, intrinsically, enforcing a key-note seems to have been clearly known. Later, the key-determining significance of the *drone* was known in practice, though not in theory, by the pipers playing the bag-pipe, and by Monteverde when he "discovered" *the dominant-seventh chord.*[9]

Rameau's mistake in substituting two modes for the ancient seven modes was an error of over-emphasis only,—of over-emphasizing *one* important, but not exclusive, method of enforcing either one of two notes (A or C) as the key-note. His mistake has resulted in denying to the composer the freedom of further inventions, and has thus hampered musical progress.

9 Probably few musicians have thought of the drone and the dominant-seventh chord as being almost the same thing. But in both cases the G is emphasized and thus reacts on the C, helping to secure for C the dignity of being the key-note. Compare also footnote 7.

Rameau's Theorem consists in the following statement with which the student is now, after solving problems 40 to 52, perfectly familiar:

> IF WE HEAR, *in music in the diatonic scale, very frequently all the (three) possible combinations of tones making up the little scale* 1 — 3 — 5, *then the most naturally chosen key-note is C and the key-notes of the other two little scales are F and G.* AND IF WE HEAR *very frequently all the (three) possible combinations making up the little scale* 3 — 5 — 15, *then the most naturally chosen key-note is A and the key-notes of the other two little scales are D and E.* AND IF WE NOW COUNT THE KEYS *from C to F and G, or from A to D and E, we find that* THE SECONDARY KEY-NOTES ARE IN BOTH CASES THE FOURTH AND FIFTH KEY.

Rameau's mysticism.—What a pity that it pervades our music teaching!

This purely accidental "symmetry", that is, in truth, only the incidental fact that the secondary key-notes were *the fourth and fifth keys* from the primary key-note *in both cases*, so bewitched the mystic Rameau that he forthwith forgot that there are many other triads than 1 — 3 — 5 and 3 — 5 — 15.[10] He decreed that no music was really worthy to exist except two kinds, one kind with the key-note A and with an almost constant reiteration of the chords A — E — C, D — A — F, and E — B — G, and the other kind with the key-note C and with an almost constant reiteration of the chords F — C — A, C — G — E, and G — D — B. For these "only two rightfully existing" kinds of music he invented the peculiar terms "minor" and "major" in linguistic conformity with the—totally irrelevant—accident that when one steps from A to the third key (C), he makes a "minor" (i. e., smaller) step than in stepping from C to the third key (E). The latter step is a semi-tone "major", that is, larger. (Compare footnote 57.)

The esthetic irrelevance of the terms "minor" and "major" as *arbitrarily* referring to the span measured in cents of the *second* interval from the key-note was clearly pointed out a century ago by Goethe. Many recent writers recognize that "rhythm rather than scale expresses joy or sadness", referring by "scale" to size of the intervals. Nevertheless the music teacher, subjecting himself to language which has become traditional, still clings to Rameau's mystic doctrine that all music must be "either minor or major", that is, that there can be *only two* proper scales, A B C D E F G A with its minor Third, and C D E F G A B C with its major Third, and that all *simultaneous* tones (duads, triads, tetrads, pentads, etc.) are "dissonances", *bad sounds*, except the two combinations 1 — 3 — 5 and 3 — 5 — 15. And this mystic theory is instilled into the mind of every pupil in every grammar school and thus becomes an ineradicable dogma.

10 See the list above, problem 37; and Appendix 1.

For the composer there is no end of variety of possible scales any more than there is of games of chess,—but they are not all equally elegant, to be sure

The term "dissonance" is *undefinable*.[11] The only statement justifiable is that there are degrees of more or less fitting together. How many of those who accept blindly Rameau's dogma, instilled into them in their childhood, even know that it is Rameau's? How many of these few have read Rameau's books and know how illogically, mystically, he got his ideas? Most music teachers teach Rameau's doctrine as something self-evident, knowable by intuition: "All music is either major or minor!" *If we hear* frequently the chords 1 — 3 — 5 and 3 — 5 — 15, then indeed we can figuratively speak with Rameau of a certain quasi-geometrical "symmetry" between these two scales, the major and the minor diatonic. But it does not either by logic or legal statute or esthetic convenience follow that every composer is under obligation to saturate us just with those two chords and no others. *And if he is not*, the whole doctrine of "the two modes of all music" falls to the ground.[12] Too bad that the possibilities of music are not so simple! Or would "how fortunate!" be the keener observation?

Still further musical triads, in spite of Rameau's condemnatory terms "dissonances, incidentals, passing notes"

In problem 37 we found a large number of triads. But those within that list are far from being all which can be used with good musical effects. A

11 Compare the futile attempts of the medieval theorists at "permitting and forbidding" certain harmonies, which will be referred to in problem 59.

12 The great French mathematician, D'Alembert, in a reply to a printed letter of Rameau, almost two hundred years ago, made the following striking remarks in his *Elemens de Musique* (ed. 1766, p. 211ff.), which illustrate the attitude of the man of science and expose the spirit of mysticism which dominated Rameau: "L'Académie des Sciences . . . . n'approuvera jamais les efforts que vous avez faits pour trouver le principe de la géométrie dans le corps sonore. . . . Le corps sonore ne nous donne & ne peut nous donner par lui-même aucune idée des proportions. . . . Pour tout dire en un mot, quand les hommes seroient sourds, il n'y en auroit pas moins pour eux, des rapports, des proportions, une géométrie. . . . A l'égard des proportions, il est certain que les proportions arithmétiques & harmoniques, si souvent & si gratuitement employées par vous, sont entièrement étrangères & par conséquent inutiles à l'art musical. . . . Pour moi, Monsieur, j'ose croire que l'Art ira peut-être un jour plus loin que vous ne pensez. L'expérience m'a rendu circonspect sur les assertions en matière de Musique; avant que d'avoir entendu vos Opéras, je ne croyois pas qu'on pût aller au-dela de Lully & de Campra; avant que d'avoir entendu la Musique des Italiens, je n'imaginois rien au-dessus de la nôtre." Probably no other words than these of D'Alembert can more forcefully expose the spirit of mysticism which to this day pervades (Rameau's doctrine of) "the major and minor scales"; and no other words can more characteristically express the unchanging attitude of the man of science who refuses to limit the future of human culture because of existing theories,—no matter how useful such theories may be to a particular person at a particular moment, as the diatonic major and minor scales are (perhaps,—but is the primary teacher sure of it?) in making piano lessons fairly interesting to a little child.

few more could have been added then, if we had *combined with itself* every one of the duads, a, b, c, d, e, and f, mentioned in that problem,—thus, aa, bb, cc, dd, ee, and ff, resulting. We could thus have added under the heading aa the triad 1 — 3 — 9, under bb the triad 1 — 5 — 25,[13] under cc the triad 1 — 7 — 49, under dd the triad 9 — 15 — 25, under ee the triad 9 — 21 — 49, under ff the triad 25 — 35 — 49. There would still further be plenty of other triads to be added if we introduced further original duads (for example, 1 — 9) in addition to the six we have designated as a, b, c, d, e, and f.

Tetrads still further help us to rid ourselves of the totally superfluous concept of dissonances

The number of possible tetrads (combinations of four tone-symbols) is even more immense. Let us pick out one for examination at this moment. Let us pick out one which played a sensational role in the history of music a few centuries ago. It was a favorite of Claudio Monteverde (1567-1643) in Venice. It is the tetrad 1 — 3 — 5 — 7 in any of its 12 possible positions.

Problem 53: Enumerate the possibilities of giving expression to the tetrad 1 — 3 — 5 — 7 within the diatonic scale A B C D E F G. With what key-symbols will you then identify the tone-symbol 1 of this tetrad? Utilize the "table of spans". Answer:

Be warned against reversing your conclusion and saying that the four key-symbols which constitute the answer you just gave, must *therefore always* be represented by the tone symbols 1 — 3 — 5 — 7. For example, see if you cannot replace 1 by 9, 3 by 27, 5 by 45, and 7 by 1.

Problem 54: Suppose you want to give expression to the tetrad 1 — 3 — 5 — 7 in *three different ways*, identifying 1 not only with G (as in problem 53), but also with the other dominant, F, and with the key-note C. What is the exact number of additional (black) key-symbols which you find you have to introduce, and which of the five, Ais, Cis, Dis, Fis, Gis,[14] will be introduced? Answer:

Problem 55: The duad 1 — 3 can be expressed in the diatonic scale A B C D E F G six times even without introducing such high numbers as 243, 729, 2187—avoid them on this occasion! If in each of these six possibilities you want to enlarge this duad into the tetrad 1 — 3 — 5 — 7, you have to enlarge the diatonic scale A B C D E F G by the introduction of several additional key-symbols. How many key-symbols do you have to introduce? And which of these five, Ais, Cis, Dis, Fis, and Gis? Answer:

Problem 56: In the manner of the scale just preceding problem 24, that is, in two lines of key-symbols and tone-symbols, and using brackets over the

13 First much used by Wagner.

14 We intentionally avoid speaking of "flats" because we wish to refer only to keys on the piano, not to any *traditional theories* with which we are not here concerned.

tone-symbols whenever indicated by identity of key-symbols, write an enlarged scale which contains all the key-symbols and all the corresponding tone-symbols contained in problems 33, 48, 54, and 55, and which in addition gives expression to the tetrad 1 — 3 — 5 — 7, identifying this "1" with the tone-symbol "25" in the completed scale, and identifying "25" with the key-symbol Fis. Answer:[15]

How composition became easy,—and trivial

The purpose of giving the student the last few problems, culminating in problem 56, is especially the following. These problems aid the student in seeing the manifold possibilities of producing new and varied melodic effects. Saying "melodic" we do not pedantically try to introduce a needless distinction between harmony and melody.[16] Simultaneity, of course, is not succession, and succession not simultaneity. But we are at present concerned with tone relations which are independent of time relations and therefore conspicuous in various time relations.

These tone relations can be *more easily observed* in succession, when the peculiar roughness often resulting from simultaneity does not *distract the listener's attention*. For this reason one should often, as above, without emphasizing succession, nevertheless quite naively speak of the "melodic" effects no matter whether the tones are successive or simultaneous.

According to Rameau and to those who have adhered to his teaching for two hundred years (during which the typical melodic possibilities of the diatonic scale have been exhausted) the possibilities of producing varied melodic effects are virtually *restricted to key-shifting*, that is to "modulation" into a different key-note, chosen as far as possible from the same seven of that scale from which the modulation proceeds, and to modulating then again and again, always being limited, however, to seven notes in each new scale. *And further*—according to the dogmatism of Rameau—"chords" are always "triads", and legitimately only these two triads, 1 — 3 — 5 and 3 — 5 — 15. All other tone combinations are bastards, dissonances. These notions of Rameau have extraordinarily limited the development of music, have arrested the inventiveness of musicians, by the very fact of *making it easy* for those who lacked inventiveness to write "pretty" music in a conventional style.

In what sense our method of using just 12 (and not 24) as the divisor of the octave span limits the composer

Problem 57: If in solving problem 56 you followed the suggestion derived

15 It must not be supposed that the somewhat elaborate scale called for in problem 56 has any particular virtue. It is simply one among thousands of scales, one among thousands of groups of tone-symbols neither better nor worse, for the composer's ends, than this one. It is an arbitrary example, chosen for drill in the use of tone-symbols in relation to key-symbols. It might suit some opera score.

16 "Harmony is frozen melody" is a good slogan. "Melodiousness" ought to replace entirely the term "consonance" in musical theory.

from problem 48 and identified the tone-symbol 15 with the key-symbol A, you will, or at least you ought to, feel very undecided about identifying the tone-symbols 35, 105, and 175. Give the reason for this—very proper—lack of decision by showing (a) that 35 lies almost exactly in the middle between two key-symbols and tell their names (between B and C or between C and Cis or what?), by showing (b) that 105 lies between two and name them, and (c) by doing the same for 175.

Answer (a):

Answer (b):

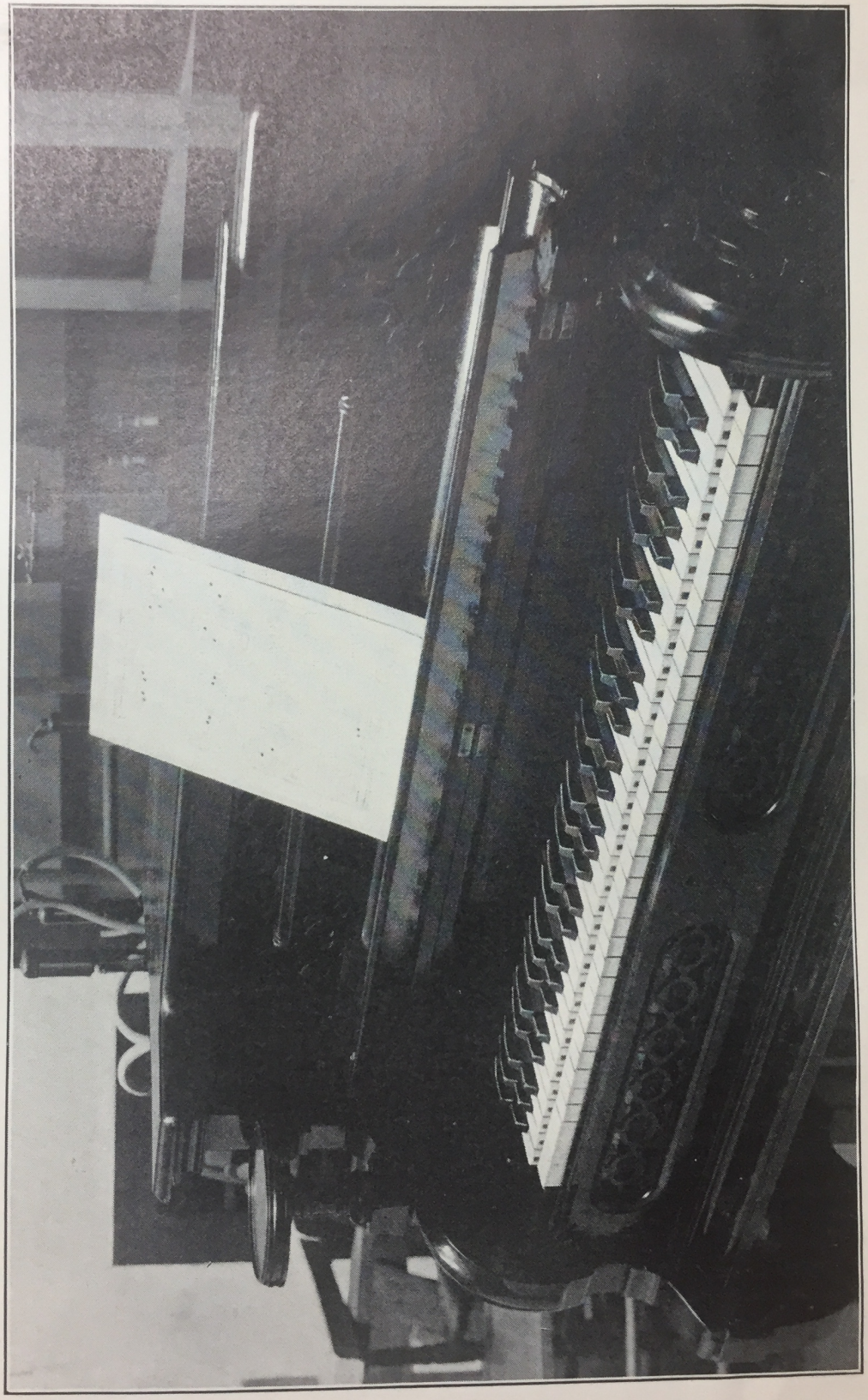
Answer (c):

You learn from problem 57 the extremely important fact that certain "melodic" (remember the very broad meaning just given to "melodic") effects (e. g., depending on 35) are not realizable on the traditional piano or organ with its twelve differently named keys and its division of the octave into twelve equal intervals. Of course, if the hearer were thoroly familiar with these melodic effects, and very unfamiliar with the others which are now commonplace to him, he might "hear" in his melody such a (to him then familiar) tone 35, 105, or 175 even when you play those tones half a semitone *mistuned* on the ordinary piano. But in all probability he will "hear" a more conventional tone whose tone-symbol *lies close* to one of the twelve key-symbols of your piano tones.

Are the grapes sour?

You may, of course, if you are a conservative composer, decide that *you do not care* to produce these effects, not easily realizable by you with your instrument, and let it go at that. But if you are more adventurous and want these effects perhaps for the very reason that by necessity they have hitherto been uncommon, how are you going to help yourself?

All that is necessary is to introduce twelve additional keys, to divide the octave into twenty-four equal intervals, and to give the twelve new tones which thereby result to the twelve additional keys of your reconstructed instrument. Such a key-board is called a quartertone key-board in order to distinguish it from the usual semitone key-board. And music using in a single piece *one or more* (hardly ever all) of these additional tones is called quartertone music or bichromatic music. Such music is no new invention, for it has existed in the orient (Arabia, India, Japan) for hundreds, maybe for thousands of years. It is not performed, however, in the Orient on a key-board instrument. And for European-American hearers, it is *by the musician of the Orient* so "unconvincingly" (or not at all) *harmonized* that Western hearers "hear" it as a "mistuned" kind of performance of *their own conventional music*. By him who desires to avoid this unfortunate result very clever methods of harmonizing will have to be used, very different from those taught in the simple theories of Rameau and his successors.

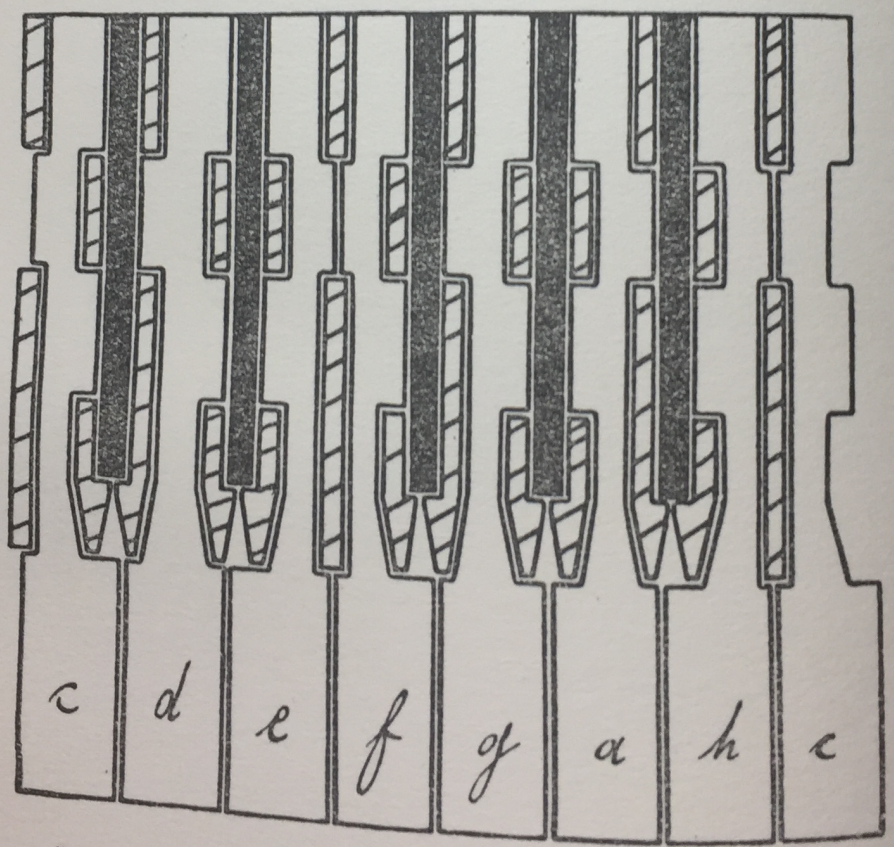


TWENTY-FOUR TONES TO THE OCTAVE. KEYBOARD DESIGN OF MAX F. MEYER

The only hope to devise such harmonies theoretically, since they are not found in our theoretical textbooks, lies in familiarity with "the musician's arithmetic". That a musician will find these new methods by a mere appeal to his auditory imagination, is probable only in case his genius is equal, if not even superior, to that of a Beethoven. Who believes that he is a new Beethoven? But "the musician's arithmetic" might help even an humbler musical spirit to compose quartertone music *which does not sound mistuned*, provided, of course, that *the listener has made himself familiar with it.* Before familiarity is established, even semitone music may appear queer, and even to a gifted musician. But familiarity is only a *conditio sine qua non.* Familiarity is not the essential factor of the esthetics of music, contrary to a wide-spread opinion. The esthetic effectiveness lies in the music itself, in its "arithmetical form", so to speak.

A twenty-four tone keyboard and a suitable staff on which to write for it

The scale called for in problem 57 (that is, 9 — 75 — 5 — 81 — 21 — 175 — 45 — 189 — 3 — 25 — 105 — 27 — 7 — 15 — 125 — 63 — 1 — 135 — 35 — 9) is a fortuitous sample of a complex scale. It happens to require for its proper execution not ony the twelve keys of the ordinary keyboard, but also four "quartertone" keys; one between B and C, for 35; one between Dis and E,

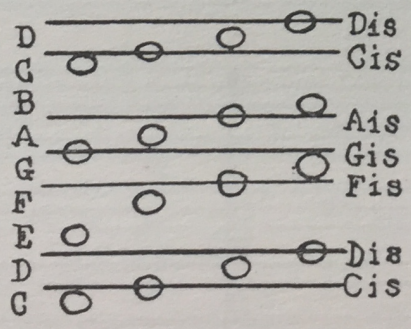


MÖLLENDORFF'S KEYBOARD. COMPARE IT WITH MEYER'S

for 175; one between Fis and G, for 105; one between A and Ais, in order to

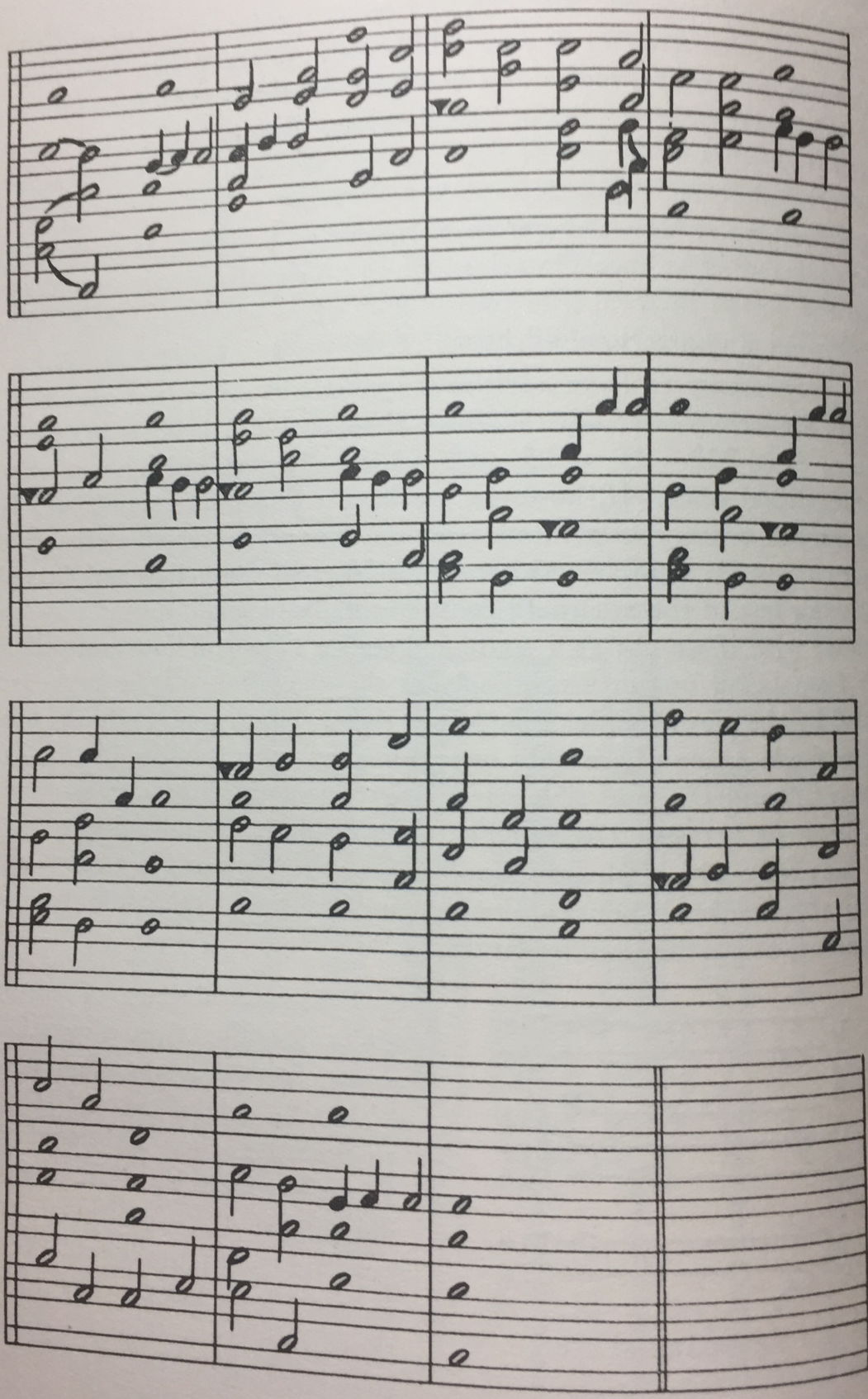
make 125 distinct from 63 and 1. Now, we are *far from* suggesting that a composer *must limit* himself to these few additional tones (piano keys) which the audience hears as 35, 105, 125, 175 in numerical notation, altho we are also *far from recommending* an overindulgence in quartertones (for mere ornamentations, glissandos, lugubrious wailing, and that kind of effects), say, a use of all twelve quartertones in a single piece. But even if a composer limits himself (to even a single one of them in one melody), he will *want to have all* the twelve additional (quartertone) keys in order to choose his pitch freely, *to transpose freely*. If we therefore introduce any additional keys at all, we ought to introduce at once all twelve and, in tuning afterwards, systematically divide the octave interval into twenty-four equal parts. (Compare illustration showing a quartertone keyboard of the present writer's design.)

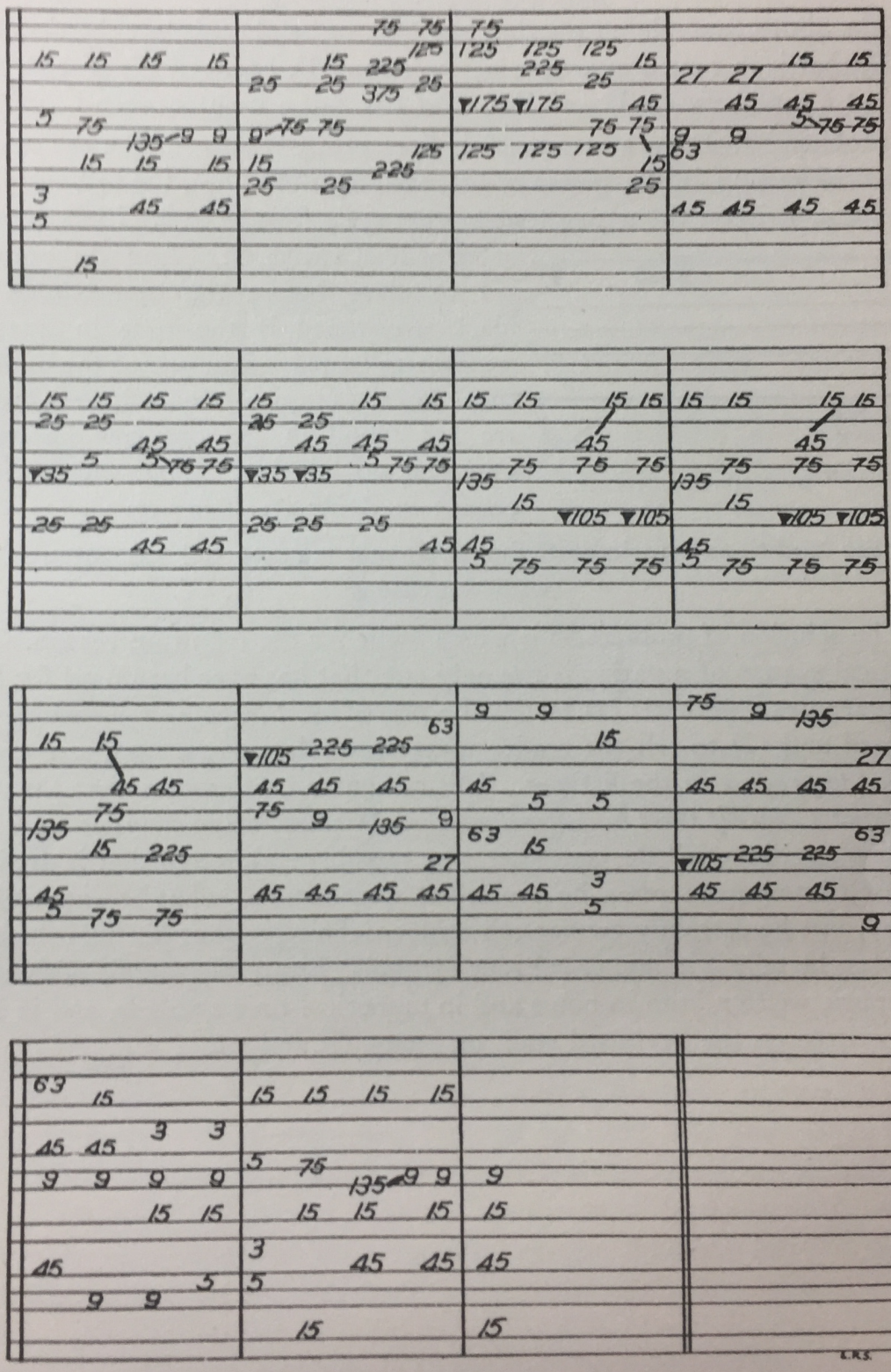
It is necessary to have additional symbols in order to write on a staff music in the scale 9 — 75 — 315 — 5 — 81 — 21 — 175 — 45 — 189 — 3 — 25 — 105 — 27 — 225 — 15 — 125 — 63 — 135 — 35 — 9, which is similar to the last mentioned scale. If, however, we have to introduce new symbols, we might as well courageously throw away on the same occasion as many of the old ones as are dispensable. As a matter of fact all the sharps and flats of our traditional, but in the twentieth century totally illogical, music staff can be dispensed with if we use as a simplified staff a system of lines grouped on the paper from below up correspondingly as the black keys of the keyboard are grouped by the piano builder from left to right. That is, we group the lines by using a single space where only one white key separates the black keys, as between Cis and Dis, and a double space where two white keys separate the black keys, as between Dis and Fis. The result is the following staff, from which anybody who is already familiar with the keyboard can learn to play fluently after five or six hours of practice (preferably distributed, for psychological reasons, over a dozen different days).

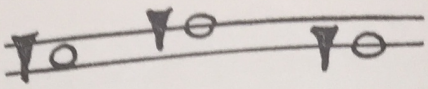


KEYBOARD STAFF

The total absence of alteration signs for the twelve usual keys *makes it easy to introduce the new alteration symbol* wanted in order to indicate the additional keys. In the keyboard shown in the above illustration the additional twelve keys are merely the tips of the original and long keys. The tones of these tips are a quartertone lower than the tones of the long keys. On the staff an easy way of indicating that the tip is to be played consists in placing a black arrowhead or triangle downward pointing before the note, in the following manner:

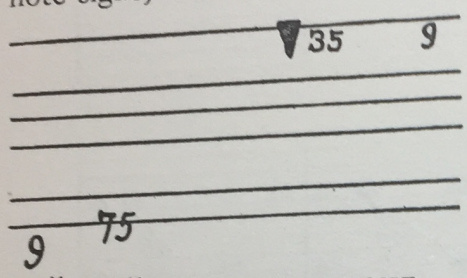



E.R.S.



The above example means that not D is to be played, but the tip of the key D, that is, a tone a quartertone lower than D, between Cis and D. Not Dis, but a tone between D and Dis. Not Cis, but a tone between C and Cis.

Problem 58: Draw a system of six lines properly spaced, like the sample. Instead of writing, in the spaces or balanced over the lines, the ordinary oval note signs, write-in the above tone-system scale from 9 to 9. Identify quite arbitrarily C with 9. And do not forget to place before the tone-symbol the black arrowhead if the tone in question requires to be played on one of the quartertone keys (a quartertone lower) which are not found in the ordinary keyboard. Can you do all this? Your answer is to consist in doing it: Answer:



"FLAT" BY A QUARTERTONE

On the traditional music staff the composer cannot tell his intentions with exactness

The solution of problem 58 is worth while for the following reason. This numerical system of notation is the only one that has ever been used for both these purposes together: (1) to tell the player *what to play* on a quartertone keyboard and (2) to tell the reader what tone relations *the composer exactly intended to produce* in the listener. This of course presupposes that the composer *knows himself what his intentions are*. Without being familiar with "the mus cian's arithmetic" the composer may possibly—although that is doubtful—himself know even when he prescribes quartertones what he wants to tell thereby, but he certainly cannot tell others definitely what his full intentions are unless he tells it in numerical tone-symbols. Study the sample of quartertone music written both in notes and in numerical tone-symbols, and in both cases written on the keyboard staff, on pages 60 and 61.

The faburden illustrates how theory, therefore composition, therefore practice, and therefore again theory (closing the circle) depend on the nature of the keyboard and the staff

During the fourteenth century the seven white keys, without any black keys on the organ keyboards, still furnished the only key-symbols in which educated musicians, that is, church musicians, could think about their music. A method of three-part singing (and organ playing) then came into existence in Italy, which is known in the history of music as faburden, faux bourdon, falso bordone. (See appendix on the Theory of Chords). In those days it was customary to name the intervals of a chord neither with reference to a *key-note* chosen for the whole music piece nor with reference to the lowest tone of each chord as in so-called "figured basses". The intervals were always named *each separately* by counting from *its own* "lower" to *its own* "higher" key on the purely white keyboard. For example, B ↗ D ↗ G was said to be a combination of a Third and a Fourth, and B ↗ E ↗ G was also said to be a combination of a Third and a Fourth. Whenever it seemed desirable the statement was added that the Fourth or the Third was the lower, or the higher interval, but in many discussions of that day such a statement was deemed superfluous, and the above two examples were then regarded—in this or that theoretical respect—as equivalent. One might compare this attitude of the musicians with that of a cook toward the recipe for a soup. In rare cases it is necessary to put the ingredients in a certain order into the kettle. But in general such a statement is deemed superfluous.

This identification *in name* of what is musically (psychologically) not identifiable, may seem strange, but it is no more strange than the fact that many *modern musicians* discuss problems exclusively in *key-symbols* instead of in tone-symbols, never having made themselves familiar with "the musician's arithmetic".

The rule for faburden singing (to-wit, in three voices, of which the lowest was called the "chant") was easily learned by the music pupil of that day (and was of course invented and adopted for that very fact of its ease in memorizing and execution). It can easily be learned by the present reader if he will remember that the intervals are named *without any reference* to any key-note or to any "bass", but each interval for itself, and that the scale is A B C D E F G with the emphasis (key-note) on any of the seven and with unlimited octave continuation. The medieval rule of faburden then is this:

"Let the upper interval always be a Fourth and the lower interval always a Third, *except* that in the final harmony (and preferably in the initial harmony, too) the upper interval should be again a Fourth but with the lower interval being then a Fifth, and with the further—and more far-reaching—*exception* that tritone intervals[17] are contrary to 'good' theory, and that therefore

17 Either F ↗ B or B ↗ F was called a "tritone".

the chant tone D[18] must always have the interval D ↗ A on it, and the chant tone B even at the start or the end must have the interval B ↗ D on it."

You easily see that the upper interval is easily remembered, being *invariably* a Fourth (in the medieval sense above explained), and that the slightly *more difficult* task of the pupil consists in memorizing what the lower interval must be in order to suit the theory of the medieval teachers.

If this medieval theory to us moderns seems folly, yet there is method in it; and the method is based on the (purely white) keyboard existing at that time,[19] and it is kept sane (that is, sanely limited) by the very limitation of that keyboard. The resulting music is not bad. As soon as we substitute, for the production of twelve equally tempered intervals, a more *logically constructed* keyboard made up of *twelve* keys, the faburden rule appears in its true light as an *only accidentally* useful, purely empirical, not logically derivable rule. And there, if we have enough courage to run counter to "vested interests", we might add the following question: Are not most of the modern rules which our music pupils learn under the name of "theory", of the same character as the medieval faburden rule, that is, pseudoscientific, purely empirical? But our theory teachers have learnt them in their youth and pass them on to their pupils. After all, they do give their pupils, in exchange for the money received, the satisfaction of imagining themselves miniature Beethovens. "The Musician's Arithmetic" does not even attempt that — no more than a general arithmetic attempts to be a guide through the stock exchange.

Problem 59: Here is a plain example of the "double discant" placed above the "chant" for producing the faburden effect. In the table below (which gives all the permitted harmonies, the mediate harmonies, in the body of the music, as well as the final-initial chords) *fill in the blanks with the tone-symbols* which are *most likely* to correspond to the given key-symbols. Let yourself be guided by the *smallness* of numbers, as defined in previous examples. Answer:

	Mediate		*Final-Initial*	
	A ↗ C ↗ F	15 ↗ 9 ↗ 3	A ↗ E ↗ A	15 ↗ 45 ↗ 15
	B ↗ D ↗ G	_____	B ↗ D ↗ G	_____
	C ↗ E ↗ A	_____	C ↗ G ↗ C	_____
	D ↗ A ↗ D	_____	D ↗ A ↗ D	_____
For-bid-den	D ↗ F ↗ B	81 ↗ 189 ↗ 135		
	E ↗ G ↗ C	_____	E ↗ B ↗ E	_____
	F ↗ A ↗ D	_____	F ↗ C ↗ F	_____
	G ↗ B ↗ E	_____	G ↗ D ↗ G	27 ↗ 81 ↗ 27

18 The "chant" was the "lowest" voice in the church singing of those days.

19 This may seem a statement which is historically incorrect. Let us admit that it is

Playing the faburden harmonies on the piano or organ will convince any one who is willing or able to listen without prejudice, that these harmonies produce a surprisingly good esthetic effect, taking into account the simplicity of wording the rule which is followed by the "composer". The *needlessly* complicated part of the rule is only that which forbids the "tritone", whereby esthetically more is lost than gained, and which results from a preconceived bias of the medieval theorist against the tritone. But let us excuse the man who invented this rule even if his theory (traditional theory) got the better of his ear in the matter of the tritone. Even if he had been able himself to *improvise* in a more advanced manner, he could not in the terminology of his day under the prevailing traditions have invented a simpler *rule* which permitted every (even a dull) pupil to "compose good music". Is not the ordinary aim and ideal of our modern harmony teaching the same? Whether such "composing" is really worth while is another question. As an avocation, perhaps, yes. As a vocation, surely, no!

The tone-symbols (relatively small numbers) of problem 59 make it clear why good music could result from the faburden rule. If we follow the comparative method used in problem 49, we readily convince ourselves that it is to be expected theoretically that the faburden rule should make better music than any other rule which the musicians of that epoch, with what terminology was at their disposal, could formulate. This will help us to keep from falling into the error of smiling with disdain at the faburden music or condescendingly calling it queer. It was a practical invention of first importance, but of course not destined to govern the musical world forever.

How rarely do musicians realize the actual ambiguity of the current staff notation!

A later and therefore much more highly developed piece of many-voiced (polyphonic) music is shown by the following example from the works of Heinrich Schütz (1585-1672). It will help us to understand why in similar music performed by singers with an intermittent accompaniment the singers are often found "out of tune" when the accompanying instrument suddenly re-enters. The usual interpretation is that this is due to careless singing. So it may be sometimes; but sometimes, as we shall presently see, it is due to *careful* singing when the singers are misled by the composer's *ambiguous notation*. Unless the composer shows his intentions by numerical tone-symbols his key-symbols (ordinary staff notes) will often lead to such misunderstandings. That is one of the reasons why composers should be familiar with "the musician's arithmetic."

not literally correct for the fourteenth century. And yet it is correct in spirit. The first black keys were introduced merely to raise or lower the absolute pitch of a chant, not in order to enlarge the scale.

THE DROPPING OF THE PITCH IN ALLA CAPPELLA SINGING[20]

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV
I Sopr.	27	15	15	15	63	15	15	135	135	135	225	15	225	225	15
II Sopr.	45	45	45	45	27	45	25	27	225	225	45	3	45	45	75
Contralto	9	75	75	75	81	75	5	81	45	45		5	135	45	75
Tenor	27	45	45	45	15	45	45	45	5	135	135	135	45	135	135	135	135	15
Bass	9	15	15	15	27	15	5	27	45	45	45	5	45	45	15

Problem 60: Give to the bass note 9 in chord I the key-symbol C, and accordingly to the bass note 15 in chord XV the key-symbol A. If in chord I the singers sing exactly in tune with the accompanying organ, and if they sing the whole series of fifteen chords as above prescribed, how much in cents would the bass note 15 sung in chord XV be distant from the note A produced on the organ? Answer:

In the preceding problem it was assumed that the singers would sing the tones as prescribed in the above table. But no such presumption is justified in the reality of chorus singing when they do not have the above table of tone-symbols before them, but merely the key-symbols of the ordinary staff notation. There are at least two opportunities for a serious misunderstanding, both of them in the change from VII to VIII. Let us remember that music like the sample given is essentially a melody which has been "horizontally" compressed and "frozen" so that a number of tones (five) always stand in a "vertical" line. The esthetic unity of *each voice separately* is not very great, as everybody will admit. Therefore the intonation of each voice cannot be said to be very unambiguously determined by itself.

It follows from the last paragraph that there is not very much determination in the intonation of their tones so far as *the two sopranos* are concerned.

But *the bass* is still less determined by itself. This voice may easily misunderstand its step from VII to VIII as being a simpler combination than it is, 5 ↗ 27. Tell it below in answer (a). Of course, this misinterpretation, whatever it may be, must correspond with the right interpretation in two respects: (1) The span measured in units of S must be the same, and (2) the interval must look like the other interval for the purposes of the pianist when he plays from the musical staff notation. That is, it is not a question of substituting a totally different "note".

20 For the musical staff notation of this piece look in *University of Missouri Studies*, Original Series I, No. 1 (1901), p. 68.

The tenor may, more easily than not, misunderstand its stop as simpler than 1 ↘ 27 (in the table 5 ↘ 135). Tell it below in answer (b).

And *the alto* is especially exposed to a misunderstanding of the key-symbols of the staff. Tell it below in answer (c).

The sopranos then readily adjust themselves to these three voices.

Problem 61: What are the three likely misunderstandings hinted at in the preceding paragraphs?

Answer (a):

Answer (b):

Answer (c):

Problem 62: If now in chord I the same C which is given in problem 60 is sung, but the conditions are changed in that the singers do make the virtually unavoidable mistake of problem 61, what will then be the distance, the cumulative error, in cents between the bass note 15 sung in the last chord and the note A produced on the organ?

Answer:

The last problem must have made it clear, if this has not been clear before, that exactness of notation by the composer is of more than speculative interest,—that it is even a practical necessity in many cases where we do not at first expect to find it of any practical consequence. The falling of the voices would have been less serious if the alto had been informed that 5 was followed by 81 in Chord VIII, and that *she could not continue singing the same tone* in spite of the fact that on her music staff there appeared the same note. The conductor of the chorus should have told the tenor that his sequence from VI to VIII (putting VII out of his mind) was the very melodious interval 1-3 (reduced from 45-135), and should have told the other voices to adapt themselves to the tenor. Without a numerical transcription no chorus conductor can know this, by guessing.

If staff reform were possible, what should it be?

Our current staff notation is both pendantically cumbersome and viciously negligent. Our staff with its flats and sharps, to which nothing differential corresponds in practice, is pendantically cumbersome for the player. And it is viciously negligent for the thinker. Pedantry compels every player, in his formative period, his childhood, to learn to play while *reading that staff*, however slightly he may be interested in ever reading (the average piano pupil will never read any music in a library!) the vast musical literature which—true enough—is deposited in our libraries only in that staff. And the same pedantry discourages scientific thinking about music by *hiding the composer's intentions under that staff* which, viciously negligent, has no respect for the needs of any one whose problems require a previous understanding of the exact tone relations intended.

The time has arrived when a distinction should be clearly made between two different kinds of notation. (1) When the writer's only object is to indicate to the performer which of *the keys of a keyboard* he should strike (or what fingering he should execute on a conventional orchestra instrument in strict unison with a piano or organ), then he should use a notation which has only twelve key-symbols, and *not a single key-symbol more*. If quartertone music is in question, substitute twenty-four for twelve, but leave the statement otherwise unchanged. (2) If, however, the writer's only object is *to indicate his intended audible tone relations*, trusting their artistic execution to the good will and best ability of whoever, *after reading the writer's intentions*, may desire to execute them in play or song, then he should use a notation in numerical tone-symbols.

The customary confusion of these two objects (by the illogical flats and sharps) is unreasonable in the twentieth century. Flats and sharps are an *unnecessary complication* whenever the only object of the writer is to indicate what keys should be struck. But if audible tone *relations* are to be *exactly* indicated, flats and sharps represent a method antiquated because it has become *too simple*, a method which by the intellectual and honest composer of music should be considered long (centuries) outgrown.

If the second object is sought (more or less) independently of the first, the use of flats and sharps is indefensible because it is contrary to the ethical principle that written symbols as well as ordinary spoken language should, if we possess symbols which are capable of doing it, *unambiguously* express our intentions, unless the very ambiguity is a sport or game in which we wish to indulge.

If the first object is sought, (more or less) independently of the second, the use of flats and sharps is indefensible, because it is contrary to all principles of economy. Viewed in the long run, the profit in time-saving which will accrue to future generations far out-balances the temporary inconvenience of the generation adopting a new method. The simple lines of the keyboard staff will take the place of sharps and flats in indicating what keys are to be struck. In the case of quartertone music, the addition of a single[21] alteration sign will serve to indicate the intention of the composer.

Incidentally, but not essentially, a quartertone keyboard can be used to give a glissando effect

If the composer wants to use a glissando effect, he ought to say so. The violinist then knows what to do. If the composer merely wants *the pianist to imitate the glissando effect by using quartertones* (provided the player has a quartertone piano) the composer ought to say that he writes "quartertones

21 It is difficult to understand why, in our scientifically enlightened century, anybody (Willi Möllendorff, for example) should want to burden us with the sight of four different (quartertone alteration) signs. Compare footnote 22.

only to imitate the glissando effect". He should never leave the reader guessing as to whether a quartertone is prescribed to produce a glissando effect or is prescribed as a "singable tone", that is, not as a siren-like howling, but as a tone *preferably removed by more than a quartertone interval from either neighbor* (see Appendix 7) and having a definite numerical relation to another tone. To let the reader guess would be on the part of that writer a lack of intellectual courage, a species of dishonesty of which no composer should make himself guilty, only because, let us say, quartertones have become the fashion. "The musician's arithmetic" will help him to profess his intentions so that the public can understand them, if a "singable tone" is wanted.

The present book shows that the acquisition of the use of numerical tone-symbols is not a great task. Its acquisition requires no more intelligence or time than the acquisition of the multiplication table. And the use of the keyboard staff can be learned by an already trained musician in less than a dozen hours of practice, and by a beginner in no more than a few weeks. Flats and sharps can be defended only by him whose guiding principle in life is "It was good enough for father, and it's good enough for me."

When we previously in this book (on page 62) spoke of quartertones, we introduced them as necessary in order that certain tone relations, e. g., between 35 on the one hand and various other number symbols on the other hand, be less likely to be misunderstood by the listener. We have just pointed out—or "admitted" if the reader prefers that verbal expression—that quartertones might also be used to imitate on the piano the glissando, although the glissando is possible with perfection only on instruments where the player can slide, such as the violin, the trombone.[22]

We can add a few other possible uses of quartertones. One might use them to give the impression of an unexpected rise or fall (by a quartertone) in absolute pitch of a piece which without such a rise or fall can be played on the usual twelve tone keyboard. Such an impression would be like that produced by mischievously manipulating the speed regulator of a phonograph. It is even more doubtful if this possibility of musical art (?) is worth the trouble and expense of doubling keys and strings of the ordinary piano, than is the possibility of better imitating the glissando.

Another possible use of quartertones on the piano would consist in introducing intentional ambiguity in the manner of the Siamese, so that of two hearers, *who pay their money*, although sitting side by side each could *take his choice* as to whether he wants to "hear" a major or a minor chord while the orchestra plays a neutral third, as above explained. It is very doubtful

22 I have the impression that the quartertone chord successions in Willi Möllendorff's "Fünf Kleine Stücke für das bichromatische Harmonium" (Leipzig, 1917) are interesting through their glissando effects, and not because they are samples of "singable" melodies. Sing them and convince yourself!

if our western audiences care enough for this privilege enjoyed regularly by the Siamese.

Quartertones could also be used for increased variety of ornamentation, like apoggiaturas. For those people who attend concerts in order to enjoy the display of brilliant technique rather than to enjoy the tone relations of singable music, this argument may be a strong one. Leave it to them!

A quartertone could be used for "sharpening" what is usually called "the leading note". This advantage does not seem important enough to justify the expense of enlarging the instrument. The psychology of the leading note is described in some detail in Appendix 10, for the information of those readers who understand only imperfectly what the requirement of sharpening the leading note is based on.

After all, there seems to be only one real justification of quartertones: When available, they permit, but only now and then after many measures free from them, to produce on rare occasions certain melodic effects which with equally little ambiguity could hardly result in the hearer to whom it is all (without exception) offered on a twelve-tone keyboard piano.

Quartertones—their limitations rather than their privileges

In discussing the use of quartertones in music, let us always discuss both aspects of the question, that is, the limitations of the use of quartertones as well as the possibilities of their use. If we forget the former aspect of the question, we are apt to become too enthusiastic over the possibilities of their use. And then, unless we have thought out in a rather concrete way, as with our number system, these possibilities of quartertones, it is only a small step to beginning to speak of "revolutionizing" music for the benefit of future generations. This step has been taken on numerous occasions by Julián Carrillo. We may admire Carrillo for his courage and enthusiasm, in proposing his "theory of the tone thirteen" and doing missionary work for it. But we can hardly follow him in calling that "a revolution of music".

I express my conviction as a psychologist, not as a musician, when I assert that music will never be revolutionized. The reason is a psychological, not a musical one: Our modern music has already developed, even with the twelve-tone keyboard, to such enormous complexities that the demands it makes on the psychological capacities of the listener are too often unrealizable, except for a handful of human beings who are geniuses in listening ability. Music which becomes the mysticism of *a few geniuses per generation*, ceases to be a subject of inquiry for the man of science, who has to serve, not a few mystics, but *mankind*. For this reason, we deny, as psychologists, the possibility of revolutionizing music.

It is a minor matter when we criticize Carrillo for his choice of the term "the tone thirteen" (el sonido 13, as seen in the reproduction, p. 122 below, of one of his advertising sheets). If we add to twelve tones further tones, the first

one added must indeed be the thirteenth. But the emphasis on the mere word "thirteen" *hides* the most important result of any scientific study of music. Did we not get the result that if we enlarge the keyboard, we must logically and economically enlarge it at once to *twenty-four* keys? Carrillo, as a matter of fact, lets his enthusiasm carry him immensely beyond thirteen tones, so far that in actual practice he tries to introduce nearly a hundred tones per octave. His arguments in favor of making such a "revolution" go under *the term of his choice*, "el sonido trece," are so confused that an attempt at criticizing them is discouraging from the start.[23]

Problem 63: Turn to the two-page sample of quartertone music just preceding problem 58. Make a study of the tones composing the last beat of the second measure and the first beat of the third measure. You see them here below in the form of a scale, with two numbers missing. (You remember, of course, that we have defined a scale as any systematic arrangement of the tones composing a piece or part of a piece of music.) Copying them from the sample, add the two numbers missing; and in the vacant line below fill in the spans in cents. (You may reduce all the numbers in order to find the span more readily.)

125 ↗ 175 ↗ ——— ↗ ——— ↗ 75
F ——— ▼B ——— Cis ——— F ——— Gis

Problem 64: Now combine the tones of the first and second beat of the seventh measure with those of the scale obtained in problem 63. Fill in the seven blanks in the first line with the proper numbers. Then in the vacant line below fill in the spans in cents.

75 ↗ ↗ 45 ↗ ↗ 125 ↗ ↗ ↗ 175 ↗ ↗ ↗ ↗ 75
Gis — A — B — E — F — Fis — Gis — ▼B — Dis — E — F — Gis

Problem 65: State clearly and convincingly why the scale obtained in number symbols as the result of problem 63, can be executed without difficulty on an ordinary twelve-tone keyboard. Then state why the scale of problem 64 requires a quartertone keyboard. Show by number symbol how ▼B (flat B, in the quartertone sense) would be inevitably misunderstood by virtually every hearer on the ordinary piano; that is, what number symbol, in a theoretical description, would then take the place of 175?

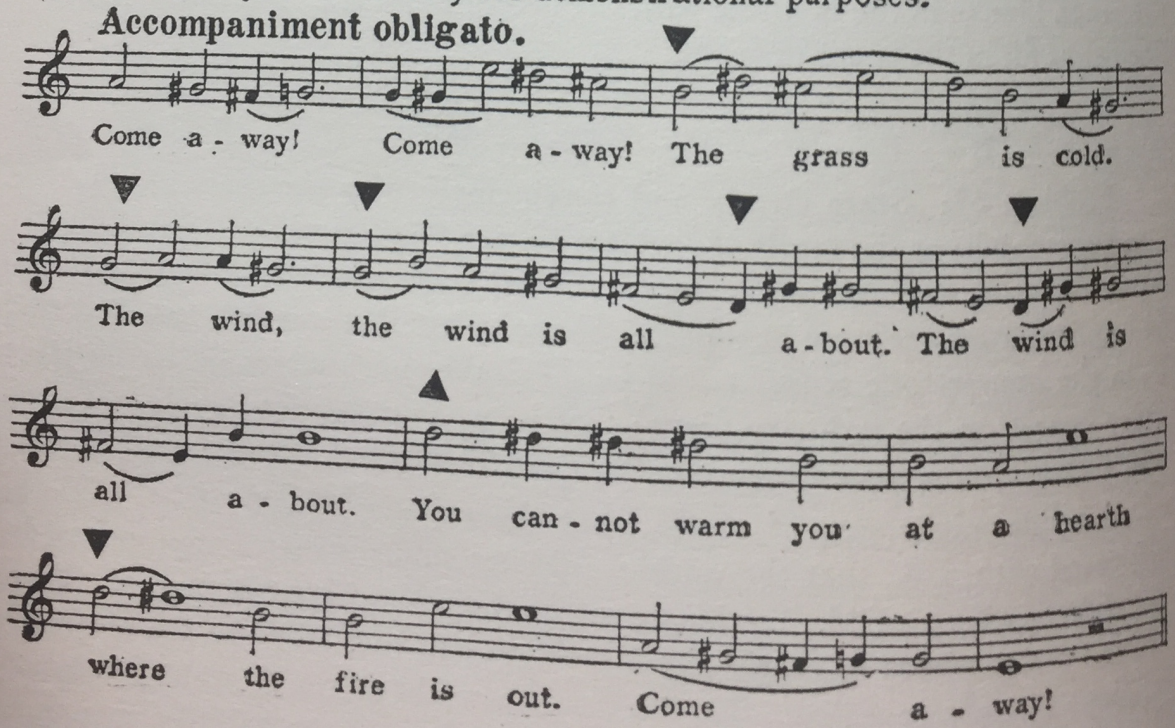
A cynic might say, of course, that the melodic effects of 175 might be dispensed with, i. e., that the world would not suffer if they were replaced by the melodic effects of 45. To this we may subscribe without hesitation. There are too many pieces of music already composed, just as there are too many religions already revealed. But the restless world seems to want constantly something new. And the individual who has formed the habit of something

23 See, however, if you are interested in such criticism, the article by Jesus C. Romero, "La Música Bicromática, el Teclado para su Ejecución y la Pauta en que Escribirla," in *La Antorcha* (Mexico, Enero de 1925), pp. 7 ff.

new or has been converted to a new faith, refuses the old, at least for a while. That is a fact which it is not our business to try to abolish.

Problem 66: Turn again to the two-page sample of quartertone music just preceding problem 58. You are familiar with the fact that songs are usually composed by first inventing the singable tune fitting an already existing bit of poetry, and then writing a suitable accompaniment. I remember having read, years ago, probably in a German social-democratic newspaper, that the last of the Hohenzollerns became a composer of fame by merely writing the words and the tune, hiring afterwards a professional musician to write the piano accompaniment needed to introduce this "song of the sea" into the homes of his loyal subjects. There is no reason, however, why one should not proceed the other way, backwards from the harmonies—to the tune—to a suitable poetry. Below you find a song which has been "composed" simply by selecting from each of the successive chords of the aforementioned sample one tone. The result is a "quartertone song". Your task consists in taking a sheet of paper, drawing the lines of the keyboard staff, and writing on it the numbers which represent the song, which is given to you in ordinary notation, just as the right half of the sample, above on page 61, of harmonies shows on the keyboard staff merely properly placed numbers. Do it now! Write the song in number symbols on the keyboard staff!

Words from a poem by Rose O'Neill.
Music written by M. F. M. only for demonstrational purposes. *



* The sign ▼ indicates lowering of the note by a quartertone.

Composing literally is a synonym of putting together, constructing, synthesizing. But in the present case it has become a synonym of breaking up, analyzing out. Last of all: words taken from a poem by Rose O'Neill have been fitted to the song. There is no logic which requires the reverse procedure, even though the reverse procedure, as we all know, is by far the more common one, is probably the only one which is taught in music schools by music teachers.

A DUET IN TONE-SYMBOLS

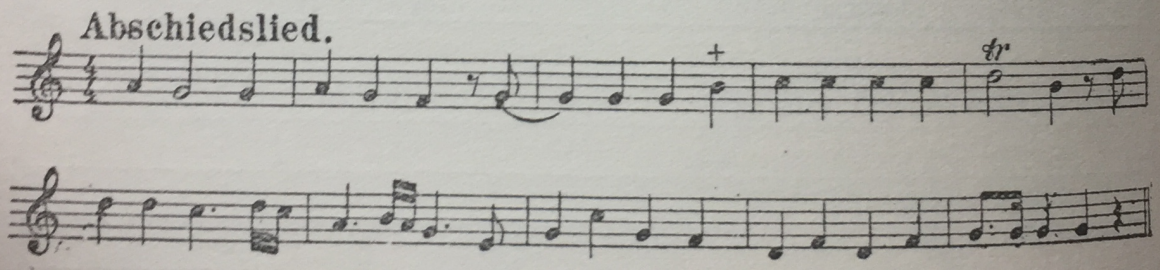
5			135	9			45	3	45				
3	15				63	9		5	75				
135	75			5		75							
45	15	105	25	45		15	225						
45					9		5						
75	9	135	27	63		15	3						
5				9									
75	9	135	27	63		15	3						
5	9	75	5	45	27	45	27	15	3	15	45		
3	15	135	9	63		9	5	75					
15	27	105	25	45	75	5	15	27	105	25	45		
45	15	105	25	45		15	225						
45	15	27	45	9	45	5							
75	9	135	27	63		15	3						
45	15	27	45	9	45	27	45	3					
75	9	135	27	63			15						

Problem 67: A duet is given you above in numbers. It is written in eight lines, like the eight verses of a stanza. (After the eighth line, if executed by a player, the first four are expected to be repeated for greater fullness of the esthetic effect.) It will be your task to rewrite the eight lines on the keyboard staff, not in numbers, however, but by properly placing the usual ovals

designating whole, half and quarter notes.[24] We advise you, for ease of discussion within the confines of the present publication, arbitrarily to choose 5 as identical with A on the staff. Do the writing now, and if you have a quartertone instrument, play the duet on it!

If this means "revolutionizing" music, then an unsuspecting hearer ought to be "terrorized" by it. Play it on a quartertone instrument to an unsuspecting listener, calmly ask him if it is not perfectly "trivial," and find out if he does not readily assent.

Problem 68: A Japanese song, called Parting Song, is given in ordinary musical notation. In the third measure there is a B which by Abraham and Hornbostel,[25] who took a phonographic record of this song, is marked with a cross, whereby they indicated that it was really a tone midway between B and C. In keyboard staff notation we shall, of course, consistently applying the same method once agreed on, write it as ▼C, as a flat C in the sense of "flattened by a quartertone". We have here a piece of quartertone music which is comparatively rare in so far as it contains an interval of just one quartertone, not a multiple of quartertones as the directly preceding interval G ↗ ▼C, which has a span of nine quartertones. Take now a sheet of paper, draw on it the lines of the keyboard staff, and then write on it in numbers the successive tones of the melody, calling the A, with which the melody begins, 15, and the G, with which the melody ends, 27. Then write an accompaniment consisting of two lower voices, so that the whole could be sung as a trio. Write the numbers of the two accompanying voices also on the keyboard staff. Then play the whole on a quartertone instrument if you have one.



Of course, the Japanese, in their native style of performance, do not use such harmonies, and they understand the music properly anyhow, because from early youth they are familiar with it. But you will surely not understand it in their way unless you add an accompaniment which is such that it forces you to interpret, in listening, the ▼C as the tone which you indicated by your number, and not as either some B or some C.

24 The reader who wishes to see printed music on the keyboard staff should obtain, for example, Beyer, *Preliminary School for Pianoforte Published in Modern Notation*, Robert B. Robinson, 4243 Garfield Ave., Kansas City, Mo.

25 "Studien über das Tonsystem und die Musik der Japaner," *Sammelbände der Internat. Musik-Ges.* IV (2), 1903.

APPENDIX 1. THE COMPLETE TABLE OF CERTAIN TRIADS[26]

	I—I	I—II	II—I	II—II
ab	1 — 3 — 5	1 — 5 — 15	1 — 3 — 15	3 — 5 — 15
ac	1 — 3 — 7	1 — 7 — 21	1 — 3 — 21	3 — 7 — 21
ad	3 — 5 — 9	3 — 5 — 15	1 — 3 — 5	5 — 9 — 15
ae	3 — 7 — 9	3 — 7 — 21	1 — 3 — 7	7 — 9 — 21
af	5 — 7 — 15	5 — 7 — 21	5 — 15 — 21	7 — 15 — 21
bc	1 — 5 — 7	1 — 7 — 35	1 — 5 — 35	5 — 7 — 35
bd	3 — 5 — 15	3 — 5 — 25	3 — 15 — 25	1 — 3 — 5
be	3 — 7 — 15	3 — 7 — 35	3 — 15 — 35	7 — 15 — 35
bf	5 — 7 — 25	5 — 7 — 35	1 — 5 — 7	7 — 25 — 35
cd	3 — 5 — 21	3 — 5 — 35	3 — 21 — 35	5 — 21 — 35
ce	3 — 7 — 21	3 — 7 — 49	3 — 21 — 49	1 — 3 — 7
cf	5 — 7 — 35	5 — 7 — 49	5 — 35 — 49	1 — 5 — 7
de	3 — 5 — 7	9 — 21 — 35	9 — 15 — 35	15 — 21 — 35
df	15 — 21 — 25	15 — 21 — 35	3 — 5 — 7	21 — 25 — 35
ef	15 — 21 — 35	15 — 21 — 49	15 — 35 — 49	3 — 5 — 7

APPENDIX 2. A POSSIBLE HARMONIZATION OF A JAPANESE QUARTERTONE SONG[27]

$$\left\{\begin{matrix} 15 & 27 & 27 & 15 & 27 & 3 & 27 & 27 & 27 & 27 & 35 & 9 & 9 & 9 & 9 \\ 45 & 45 & 45 & 45 & 45 & 9 & 45 & 45 & 45 & 45 & 15 & 15 & 15 & 15 & 15 \\ 9 & 9 & 135 & 9 & 9 & 15 & 9 & 15 & 135 & 9 & 5 & 45 & 45 & 3 & 45 \end{matrix}\right\}$$

$$\left\{\begin{matrix} 5 & 135 & 5 & 5 & 5 & 9 & 5 & 9 & 15 & 135 & 15 & 27 & 45 \\ 15 & 15 & 15 & 15 & 15 & 15 & 15 & 15 & 3 & 45 & 45 & 45 & 9 \\ 3 & 45 & 3 & 45 & 3 & 3 & 3 & 3 & 9 & 135 & 135 & 135 & 15 \end{matrix}\right\}$$

$$\left\{\begin{matrix} 27 & 9 & 27 & 3 & 5 & 3 & 5 & 3 & 27 \\ 45 & 27 & 45 & 5 & 15 & 5 & 15 & 5 & 45 \\ 9 & 45 & 9 & 15 & 3 & 15 & 3 & 15 & 9 \end{matrix}\right\}$$

26 Compare Problem 37.
27 Compare Problem 68.

APPENDIX 3. THE MYTHOLOGY OF OVERTONES

To a psychologist of the twentieth century, trained in the physico-chemical and biological notions of his time, it seems almost incredible that for a couple of hundred years musicians and others interested in the science of music should have been willing to amuse themselves with such fairy-tales as that of the magical power of the overtones. Since the name of these tones is not quite in accordance with the scientific terminology of today, and was originally only a falsely translated German idiom, "die *oberen* Töne," meaning "the *upper* partials," let us call them "partial" rather than "over" tones. Rarely are physical tones produced in any other way than by the *vibrations* of masses either cut out of solids, or confined within a container if fluids, or artificially stretched if strings. In any of these three cases the mass is the readier to vibrate (after having been struck) *in partial masses*, that is, in fractions such as halves, thirds, quarters, etc., of the whole mass, in addition to the vibration of the whole mass, the thinner the volume of the whole mass, the thinner the volume of the mass relative to its length. Proof: compare a thin string with a thick string, or the air in a trombone which is a hundred times as long as thick with the air volume scarcely a dozen times longer than thick in a flute.

The older (nineteenth century) musical psychologists indulged in mythologies like this: Partial tone 5 often appeared together with partial tone 4, *therefore* the listener is pleased if 5 should follow 4 or the reverse; but he is left cold and unsympathizing if 5 should follow 23, which is, for the mechanical reasons mentioned, a rarer companion. Since, however, no one ever reacted to overtones until, since *a few centuries ago*, it became the hobby of a few enthusiastic physicists to call the attention of a few other physicists (their students) rather artificially to the audibility of these weak accompanying tones, one would think the conclusion inevitable that music is only a few hundred years old and is an esoteric amusement of a few "scientists". The truth is, that music is thousands of years old and is, in its essentials, the universal possession of mankind. Of course, to those who are willing to indulge in speculating about the unlimited power of "unconscious experiences", such a realistic, objective and critical argument will not appear convincing. Neither is this book of drill problems written for them. The psychologist always has a hard time saving himself from his "friends," the dreamers.

A mythology no less strange than the "overtone" mythology was offered to the world about twenty years ago by a physicist, now deceased, who happened to be professionally much interested in the internal reverberations, the so-called acoustics, of lecture halls, theaters, churches, etc. Such reverberations are apt to bring about a confusion of sound quite contrary to the wishes of the architect. The physicist in question caught the notion that all mankind might be derived from "a caveman", Adam. When Adam, having retired from a day's work into his cave, sighed, or howled, or screamed, or roared, and

accidentally one of these sounds, fitting the volume of this particular cave, reverberated strongly, not all other sounds issuing from his throat reverberated strongly, too, but only particular sounds. (They all had to be partials belonging to the air volume of the cave.) That astonishing discovery interested Adam so much that, even when he tried to amuse himself in the open field or in a mere tent or hut, he chose to howl only in the intervals which he had "learned" in his cave. And thus musical intervals became a part of human civilization. This is the "caveman" mythology.

In the light of modern science, driving out the obscurities of former mythologies, it seems obvious that because of the "stereochemistry" of our neurons tones *variously* paired call forth "reaction patterns" of lesser or greater, that is, *varying* similarity. And thus music can become a counterpart of the dramatic aspects of life. Is not life "dramatic", that is, tragical or indifferently strenuous or comical, because of *the lesser or greater similarity, or call it congruity*, of the kaleidoscopic events passing in review? Durations or intensities also play their role in music, especially in martial, choric and otherwise applied music. But it is mainly for *biochemical reasons* that music can represent the esthetic dynamics, so to speak, of life. It all lies hidden in the mere *chemistry of our nervous tissue*. Let us look forward to the lesson biochemistry will give us in the future, and meanwhile be patient.

APPENDIX 4. A BRIEF HISTORY OF THE THEORY OF THE DIATONIC SCALE

During a period of time probably including the years 550 and 500 before the Christian era, there lived in the eastern Mediterranean regions inhabited by Greeks a rather legendary personage called Pythagoras. He was famous for the readiness with which he made observations of what happened in the world, inanimate, animate and human, reflected upon these observations and communicated these reflections to his numerous pupils by whom he was almost worshiped. Among other things the vibration of strings was studied by him and his disciples. We must not imagine that they had a very definite conception of vibration frequency when such frequency was at all high. They did not possess the physical instruments necessary to measure the vibration rate even in sounds, which is nowadays among the lowest rates with which we deal in physics. What was known to the Pythagoreans was in the main only the fact that definitely recognizable musical intervals correspond to definite differences in the length of a string whose tension (caused, for example, by a weight hung over an edge) remains the same. We now know perfectly all these phenomena and prefer to speak of vibrations rates, rather than of various string lengths, since the former concept includes also those sources of tone which are not strings. Let us here simply speak of ratios without referring by the terms of the ratios to this or that.

Pythagoras, about 550 before Christ. Pythagoras, we may then say, discovered the musical significance of only two ratios, 1:2, the ratio belonging to an octave (in modern terminology), and 2:3, the ratio of a fifth. By means of these two ratios he succeeded in explaining the intervals of *the diatonic scale* and, of course, took it for granted that only the intonation thus determined was the correct one. For this latter conclusion we cannot blame him, since nearly all other theorists down to (and including to a considerable degree even) Helmholtz have acted likewise with respect to the scale intonation required by their theories. Virtually all *took it for granted that there was only one correct* intonation of the diatonic scale and tried to discover, by theory or experiment, what that one exactly was.

The Pythagoreans, in order to lay down the tones of the diatonic scale, proceeded thus. They started from one tone of the diatonic scale, let us say F, and pointed out that another tone of the same scale, in the present case C, could be defined by the ratio 2:3, which, as we remember, corresponds almost exactly to the span of seven equally tempered semitones, i. e., the interval of a fifth in the terminology of musical practice. Taking another step of seven semitones from C, G is reached. By further steps of seven semitones we reach D, A, E, and B. Representing this Pythagorean theory by our familiar symbols which identify all octave tones, we may say that the Pythagorean definition of the diatonic scale is the following:

C	—	D	—	E	—	F	—	G	—	A	—	B	—	C
3		27		243		1		9		81		729		3

There are to-day musical theorists who like the Pythagoreans accept this definition of the diatonic scale as the (i.e., the only) correct one.

We saw that in accordance with the Pythagorean definition of the diatonic scale the different tones are represented by successive powers of 3; so that, in our scale above, B is represented by 729, the sixth power of 3. If we proceed in the same way, we reach by further steps of seven semitones each the following tones transgressing the diatonic scale: Fis = 3^7, Cis = 3^8, Gis = 3^9, Dis = 3^{10}, Ais = 3^{11}, Eis = 3^{12}, etc. Now, since in musical practice Eis and F are identical and since we started from F, we may say that by making twelve times the step corresponding to the ratio 2:3, i. e., the step of a fifth upwards, we reach the same tone from which we started,—or rather, we reach a tone which is just about seven octaves higher than the tone from which we started; for twelve steps of seven semitones each (fifths) must be equal to seven steps of twelve semitones each (octaves). This fact that thus we can return to the starting tone by taking twelve fifths upwards and seven octaves downwards is usually referred to by the phrase "the cycle of fifths". It was well known to the Pythagoreans.

Aristoxenus, about 350 before Christ. Aristoxenus, a disciple of Aristotle, found by computation and emphasized against the Pythagoreans that *the cycle of fifths is slightly imperfect*; that, in order to reach the starting tone exactly, each of the twelve steps must be slightly smaller than the one corresponding to the ratio 2:3. We can easily comprehend this from our table of spans, which gives the step 1 ↗ 3 as equal to 7.02 equally tempered semitones. If each step leads us .02S too far, twelve steps lead us .24S too far. That is, the tone reached by a cycle of fifths is almost one fourth of a semitone higher than the starting tone. Aristoxenus reasoned—rightly or wrongly—that the tones of the diatonic scale ought to be defined in such a manner that by going thru the cycle of fifths *the starting tone was reached exactly*; that is, that each step of a fifth ought to be .02S less than the perfect interval 2:3. Then, of course, each step is simply equal to seven equally tempered semitones exactly, and the result is the diatonic scale of modern practice:

C	D	E	F	G	A	B	C
OS	2S	4S	5S	7S	9S	11S	12S

Aristoxenus is therefore *the inventor of equal temperament*, not in musical practice, but in musical theory. In musical practice Bach had immensely more influence than Aristoxenus. However, scarcely anyone believes nowadays that the equally tempered scale can be made the basis of an all-including *theory* of music, as Aristoxenus seems to have thought, and as some modern Aristoxenians think.

Ptolemy, second century after Christ. During the succeeding centuries the theory of the scale (or scales) became an exceedingly complicated mass of details, which is of little interest to the scientist, however interesting it may

be to the collector of philosophical curiosities. However, during this period a new thought of fundamental importance gradually gained adherents—the suspicious thought that Pythagoras made a mistake in restricting tone affinity to 1—1 and 1—3. This suspicion reached its full development in the musical writings of the famous mathematician of Alexandria, Ptolemy, most widely known as an astronomer, the expounder of the system of the planets called after him the Ptolemaic. According to Ptolemy *any relatively small number* can be used for the definition of *musical* tones. Thus we see him using all the symbols which we have admitted in our axioms 1—5, 3—5, 1—7, 3—7, 1—9, 5—9, 1—15, 5—21, 7—9, but also many which we have rejected, for example 5—11, 3—13, 15—31, etc.

There are two objections to the use of these symbols. First, Ptolemy never seems to have made any musical experiments (we really could not blame him for that!) in order to decide what symbols of tone affinity should be admitted or rejected, but seems to have felt satisfied with the purely arithmetical consideration that the numbers used were relatively small. Secondly, there can be little doubt that he thus introduced into the theory a greater number of axioms than are needed. We have seen that not all directly succeeding tones have to be directly chemically affinitive. They may be indirectly affinitive, affiliated; as in the case of 5—21 (one semitone) both tones can without doubt directly succeed each other in a melody provided a tone affinitive to both, say 3, has been heard shortly before or will be heard afterwards. The admission of the symbols 5—21 for affinity might therefore be experimentally warranted, and yet theoretically superfluous. The same is true for 9—35 (half a semitone). Instead of developing the concept of indirect affinity, which we might call affiliation, but stopping with larger prime numbers than 7, as we did, the Ptolemaic theory defines certain intervals by such symbols as 15-31, being satisfied by the consideration that the prime number 31 is not a very large number. On the basis of our axioms, 31 is altogether too large a prime number since in affiliation, i. e. indirect affinity, numbers higher than 7 seem unnecessary as factors, for our symbols seem to be able theoretically to take care of all existing music. And no scientific theory ought to be needlessly complicated.

Summarizing, we may say that Ptolemy was right in recognizing that the theoretical restriction of tone affinity by Pythagoras to 1—1 and 1—3 was unwarranted, but that Ptolemy was far too liberal in accepting any ratio which fairly represented the interval to be defined by relatively small numbers, without any low limit to the prime numbers entering as factors. All the later theories, including our own, will be found to stand somewhere between the extremes of Pythagoras' closeness and Ptolemy's liberality.

Zarlino, 1517-1590. The attempts of the theorists at finding a middle road between the extremes just mentioned led in the sixteenth century to the adoption into the theory of music of the diatonic scale 3—27—15—1—9—5—

45—3 by the Italian composer and writer Zarlino. Comparing this scale with the Pythagorean scale we notice a difference in the definition of three tones, A, E, B, if the key-note is 3 and is called C. These are represented in the Pythagorean scale by the large numbers 81, 243, and 729. Zarlino observed that much smaller numbers could be substituted by simply being more liberal than Pythagoras to the extent of admitting the prime number 5 as an arithmetical factor. Therefore he did admit 5 as a factor—not on any experimental evidence of the rightfulness of its claim, but exclusively because of the smaller product-numbers thus obtained, compared with those of Pythagoras. Thus he represented A by 5 instead of 81, since $5 \nearrow 3$, as we know, represents an interval of three semitones, which is the interval between A and C. He represented E by 15 instead of 243, since $15 \nearrow 1$, as we know, represents an interval of one semitone, which is the interval between E and F. He represented B by 45 instead of 729, since $45 \nearrow 3 = 15 \nearrow 1$, as we know, represents an interval of one semitone, which is the interval between B and C. From Zarlino's time for centuries forward the meaning of the term 'diatonic scale' has been *the scale (in the key of C) restricted to the following numbers:*

C	D	E	F	G	A	B	C
3	27	15	1	9	5	45	3

Zarlino, while admitting the prime number 5 into the system, excludes from it all higher prime numbers such as 7, 11, 13, 17, etc., thus establishing a distinct opposition to the ancient system which has just been spoken of under the name of the Ptolemaic. The reason given by him for rejecting all prime numbers other than 1, 2, 3, and 5, is very characteristic of the prescientific era to which he belongs. It is no reference to theoretic economy, that is, to the economy of thought as applied to the facts known, which is the guiding principle of the modern man of science. It is *the mystery* of the number 6, as exemplified by the fact that *space* has six directions, above, below, fore, after, right, and left. Because of this mystery only prime numbers within the series one to six are admissible. The others "would fall out of space". It is well to point this out for the benefit of those musical theorists of the present time who, having been taught this theoretical scale in early childhood, have come to regard it as something mysteriously sacred, not subject to criticism, an absolute truth, self-evident and not needing any support by facts of observation. If they regard this theoretical diatonic scale as an *absolute* truth, they are in perfect agreement with its inventor; for Zarlino too regarded it as an *absolute* truth, *founded on mystery*, on the mysterious properties of space.[28]

28 Will the time ever arrive when the authors (of whom one might expect a critical and scientific attitude) of textbooks of physics for high school students will abstain in their chapter on acoustics from nursing dogma by putting before young people at this impressionable age the just mentioned scale as "the" musical scale,—somewhat as if Zarlino had brought it down from the mountain as Moses did the commandments?

Under the influence of Zarlino, who was, if not as a composer at least as a teacher, the greatest musical genius of the sixteenth century, the diatonic scale 3—27—15—1—9—5—45—3 established itself firmly as the fundamental concept of musical theory. Thus we find it recognized one and a half centuries later by Rameau, who in 1722 made it the chief basis of *his* system of harmony, far more comprehensive, but no less mysterious.

Rameau, 1683-1764. Rameau lived in an era of science. He was not satisfied with that mysterious justification which Zarlino had offered for his acceptance of this system of numbers. By a curious accident, at this very time the attention of the educated world was called to an observation in a branch of physical science which seemed to be closely related to music—in acoustics, where a series of numbers *similar* to Zarlino's 1, 2, 3, 4, 5, 6, was discovered to play an important role. A peculiar kind of cross-fertilization of ideas caused a new concept to spring from Rameau's mind: *Zarlino's scale is the natural scale because of the fact that elastic bodies naturally produce over-tones.*

A French monk, Mersenne, (1588-1648) who was interested in tone and music, observed that a string untouched but not too far from another and vibrating string, would begin to vibrate if in unison, and also if an octave or a twelfth above the latter. This was the first intimation of the existence of partial vibrations, partial tones. In 1701 the French physicist Sauveur made them evident to the eye by placing paper riders on the nodes and ventres of a string, and gave the scientific explanation of elastic bodies, when vibrating, vibrating not only as wholes but also as parts—halves, thirds, etc. The result is that any single vibrating body makes us hear a series of tones of the vibration rates 1, 2, 3, 4, 5, 6, 7, etc, in some cases up to 20 or even 30, with decreasing intensity as the ordinal number increases. Rameau, finding among these numbers the 1, 2, 3, 4, 5, 6 of Zarlino, concluded that now everything was scientifically explained. He observed that the higher partial tones are weaker and weaker. He did not need more than the first six, or even the first five only, for his theory. And so he took it for granted that the seventh and the following partial tones, because of their increasing weakness, might be regarded as non-existent. He did not see that the restriction of his attention to the *first five* over-tones was unwarranted.

While Rameau deserves much credit for the simplification of the theory of harmony and counterpoint by discovering the concept of inversion of chords (precursor of systematically omitting powers of 2), his acceptance of Zarlino's diatonic scale was a mistake much to be regretted and his "scientific" explanation of the scale was, after all, scarcely less mysterious than that of Zarlino. We remember Rameau's theorem which confines all possible music into two "modes", the major and the minor, because of a mysterious "symmetry." Compare page 52 above.

Helmholtz, 1821-1894. The present treatise has in no way followed the path marked out by Helmholtz in his *Sensations of Tone*. I agree almost with that distinguished German musician, Moritz Hauptmann, who after a careful study of Helmholtz's work reached the following conclusion: "Helmholtz's book deserves undoubtedly all the adoration which it has found, but . . . . . if it is his opinion that his book in any way touches upon the theory of music, he is greatly mistaken."

Helmholtz observed clearly that Rameau's supposedly "scientific" explanation of the diatonic scale was illogical and insufficient, and tried to improve it. But that it was a regrettable mistake of Rameau to regard all modern European music as based on the diatonic scale of Zarlino instead of regarding the diatonic scale merely as a special (however important) group of directly and indirectly affiliated tones he did not clearly observe, although he once in awhile tried it by introducing the number 7, rejected by both Zarlino and Rameau. In his celebrated book on the sensations of tone, Helmholtz accepts the since Rameau traditional view that the fundamental concepts of musical theory are *these two:* the "diatonic scale", and "modulation". Now modulation, in arithmetical terms, means nothing but the multiplication of all the number symbols of the diatonic scale by any one of these symbols or by any product of them, and the construction thru this multiplication of a new diatonic scale of the same ratios, but made up of higher absolute numerical terms. Thus, however, no new prime number is ever introduced as an arithmetical factor in addition to those contained in the primary diatonic scale, so that the whole system can never contain any prime numbers other than 1, 2, 3 and 5. The prime number 7, which we felt obligated to introduce into our axioms, is rigidly excluded by the logic of Helmholtz although he wishes to introduce it.

It is true that we find Helmholtz now and then hesitatingly considering the inclusion of the number 7. However, in every such case he ultimately yields to the tradition originating from Zarlino—and gives "a reason" for it, too. Let us quote one of these reasons, (*Tonempfindungen*, 5th edition, p. 257): "The chord C — E♭ — B♭" (3—7—21, all tones being directly affinitive) "sounds strange, but not unpleasant . . . . . However, *it does not pay to introduce*, merely because of the pleasantness of this harmony . . . . *tones into the ordinary scale which are foreign to this scale.*" What does this mean, "foreign to the scale"? Is this not identical with saying that, *because four hundred years ago the theorist Zarlino made up his mind to bar all prime numbers above "the mysterious 6" of the space dimensions from his scale, therefore the modern composer is debarred from perceiving, in imagination and actual performance, the tone relations of* 7? Has not the distinguished scientist joined here the ranks of those theorists who, since prehistoric times, have again and again declared that certain tone relations are "forbidden" in music, *not because*

of a demonstrated lack of affinities, but forbidden simply *because they do not fit the theory then in vogue*. Actual music, naturally, has always continued on its way toward further development, in spite of all theories to the contrary. If the theory "forbids" progress, the theory will ultimately be the sufferer.

Helmholtz in no fundamental respect goes beyond the views established a hundred and fifty years before by Rameau, in no way improves the musical theory of Rameau (see above the quotation from Hauptmann) except in adopting another "explanation" of Zarlino's mysterious number series stopping at six (1, 2, 3, 4, 5, 6) for the "explanation" which Rameau had substituted for the "explanation" of Zarlino. The additional explanation offered by Helmholtz, on which strangely enough his fame in the world of musicians rests, refers especially to secondary phenomena, to beats, to *roughness* of the sound in chords. And yet, musicians are not squeamish about noise, roughness of sound. They love it. Not that he objects on general principles to explaining musical facts by reference to overtones—on the contrary, he imitates Rameau in this as in all other respects; but he amends Rameau's "explanation" by adding his new "explanation" of the alleged fact that the overtones of ordinal numbers higher than six must be regarded as irrelevant. Why do we admit, as contributing to the building of the scale, only certain triads (and why only triads and not also *duads*? This fundamental problem in passed over with complete silence) and not also other triads? Because—this is Helmholtz's answer to the question—some triads are rough and some triads are smooth; and only those which are smooth enough are admitted. Only the triads 1—3—5 and 3—5—15 are smooth enough for him—and since they contain no prime number higher than 5, the theoretical diatonic scale can not contain any prime number higher than 5.

Helmholtz entirely overlooks the fact that the musician, the composer, is uninfluenced by the roughness and smoothness of the sound, so much so that, say, the roughness naturally resulting from the use of the tempered scale is scarcely ever complained of. Any person of good auditory imagery, especially a composer, has no difficulty in *imagining* the chord A — C — E — G♯ without any roughness whatsoever. But he does not on account of its imagined smoothness, treat this chord in a composition otherwise than as if he had imagined it rough. Compare, for one example, the testimony of Hauptmann as quoted above.

The possible argument that the composer, in order to get the exact esthetic effect intended, must substitute actual auditory perception for his imagery, and that thus his esthetic affection will be altered by the then unavoidable roughness, is not supported by any introspection on the part of persons of good auditory imagery. Positive evidence to the contrary is found in Beethoven, the deaf, being moved to tears by imagining compositions of his own which he had not perceived and was never to perceive thru his organs

of hearing. No musical person, that is, no one possessing vivid and in all respects normal auditory imagery, would be bold enough to *assert* that these same compositions can also move the actual hearer to tears, only because Beethoven took great pains in imagining new, unheard of, tone combinations *in exactly the roughness* in which the hearer later could not help perceiving them. This explanation of the stopping of Zarlino's number series at six, and of Rameau's prime numbers at five, by the roughness and smoothness of triads is no less mysterious than the earlier explanations offered by Rameau and Zarlino. What the science of music needs, is not so much "explanation" of this kind as systematic description.

Helmholtz, as already stated, bases his derivation of "the" scale, under the influence of Rameau, on triads of simultaneous tones, without giving any reason whatsoever *why duads, tetrads, pentads, etc. of simultaneous or successive tones should be debarred from contributing toward the building of scales.* Had he, instead of permitting himself to become a captive of Rameau's triad theory of chords, admitted no more than the possibility of the duad chord (remember the considerable number of compositions for two voices) contributing toward the building of scales, he would have stood practically on the same foundation on which the present treatise is standing, with this difference only, that he would still place an undue emphasis on simultaneity of tones. He would then have admitted that the foundation of a musical theory is, not the two concepts of "the scale" and "modulation" but *direct and indirect affiliation by biochemical affinity within the nervous system.*

There is no doubt that Helmholtz frequently, while writing his book, felt that "the scale" (the theoretical diatonic scale of Zarlino and Rameau) and "modulation" are *no fundamental concepts.* Any one reading the third part of his book, especially the fifteenth section entitled "The Consonant Chords of the Various Modes", observes that he wishes to convince himself that "the" diatonic scale is, after all, only "a" scale. Nevertheless, he is so completely captivated by Rameau's triad theory, that he cannot help regarding all "modern" music as being based on "the diatonic scale" and "modulation". Throughout Helmholtz's book these two tendencies, which we may be permitted to call the "a scale" tendency and the "the scale" tendency, are making war upon each other. But whenever the necessity arises of ceasing to straddle the fence, of accepting the one or the other for the purpose of explaining modern European music, Helmholtz invariably yields to the "the scale" tendency.

Thus, when designing, in order to experiment on modern European music played in just intonation, an instrument tuned in just intonation, Helmholtz designs it in accordance with "the scale" and "modulation", as appears from the fact that he excludes from the experimental harmonium described in his book all ratios containing the number 7 as a factor.

Surely, Helmholtz's method of deriving scales like Rameau by combining groups of three tones, triads, as if *three* were a sacred number and triads alone had therefore a right to be considered as contributing toward scales, is a serious flaw in his reasoning. The great scientist did not succeed in freeing himself of the influence of a century and a half of tradition in musical theory.[29]

Miscellaneous Theories of the Present. Among the writers of the present time on musical theory we find some who adopt the Pythagorean definition of the diatonic scale, limiting the available ratios to 1:2 and 2:3; some who adopt the Ptolemaic[30] with its unlimited supply of available ratios; some who follow Aristoxenus, believing that the application of either the diatonic or the chromatic scale will bring about a solution of all the problems of music if only the definition by equally tempered semitones is accepted. Most of them, however, with Rameau and Helmholtz, accept the view of Zarlino. But they all agree in searching for a special, limited in scope and yet fundamental, *root-like scale*; they all fail to recognize that *musical theory can not be based on a scale*, but must be based on more fundamental facts, i. e., the affinities in pairs of tones,—that any scale is merely a derived fact without considerable significance.

A peculiar puzzle with which we find these theorists endlessly contending, arises out of the generally accepted view that the minor diatonic scale of Rameau is identical with the major diatonic scale of Rameau whose key-note is three semitones higher, for example, A minor with C major. *Historically*, this can be easily understood. *Ever since Rameau* the "symmetrical" major and minor diatonic scales have commonly been defined each as the tones of three identical triads, in the former case triads of the class 1—3—5, in the

29 It may not be amiss to relate here the following experience of mine. As a student I happened to become once experimentally interested in *certain combinations* of two or three or four or more tones. Innocently I used instead of the long term "combinations" the brief term "chords". When now interested in a certain psychological effect of a certain combination of *two* simultaneous tones, I asked my teacher—a celebrated professor and, like Helmholtz, author of a book on music—a question and in asking it spoke of this combination of only two tones as "a chord," I received the advice: "You must not call a combination of only two tones a chord." Eager for further information I asked: "Why not?" and received the puzzling reply: "Because *musicians* never do call a combination of two tones a chord." From that moment I date my conviction that there was something fundamentally wrong in the traditional "science" of music, and that it would be worth while to endeavor to find it out. Science is the born enemy of pedantry. Restricting a name without reason, merely by tradition, as my teacher restricted the name "chord", is pedantry.

Some have thought that Helmholtz's work in musical theory is comparable to that of Newton in celestial mechanics. They forget that Newton did not attempt to give a new "explanation" answering the question *why* material bodies attract each other, but made their movements plain by *deducing* them logically from a few simple axioms.

30 For example, Henry T. Moore, "The Genetic Aspects of Consonance and Dissonance," *Psychological Monographs* 73, Vol. XVII, 1914. This treatise ends with the words "no limit can be set."

latter case three triads of the class 3—5—15. We have seen that thus indeed we obtain an identical succession of intervals (spans) as defined by equally tempered semitones:

Diatonic	A	—	B	—	C	—	D	—	E	—	F	—	G	—	A	—	B	—	C
Major					3	—	27	—	15	—	2	—	9	—	5	—	45	—	3
Minor	15	—	135	—	9	—	5	—	45	—	3	—	27	—	15				
Spans			2S	—	S	—	2S	—	2S	—	S	—	2S	—	2S	—	2S	—	S

It seems but natural, then, that the identification of the major scale having the key-note C and the minor scale having the key-note A should have become so general; for musical theorists, when undisturbed by practical difficulties—and this they are here since the tempered scale is the scale of musical practice—have but rarely troubled themselves on account of a few logical inconsistencies affecting only the theory, not the practice. The present inconsistency becomes clear when, for comparison's sake, we multiply all the symbols of the series marked 'Major' by 3. We see, then, that the symbols of both scales are identical with one exception, that of the tone D. Its symbol in the major scale is 81, in the minor scale 5. It is plain then, that we can *not logically* accept both these theories: (1) that the minor scale is identical with the major scale whose key-tone is three semitones higher than the key-tone of the minor scale; and (2) that the diatonic scales in major and minor are, respectively, the creations of the triads 1—3—5 and 3—5—15. One of these theories excludes the other;—my own opinion is that both of them ought to be thrown overboard, together with the very concepts of 'major' and 'minor', for the descriptive characterization of music as major and minor is entirely insufficient and often grossly misleading. I speak, of course, of musical theory; in musical practice, for example in kindergarten teaching, the concepts of 'major' and 'minor' may continue to serve many a useful purpose long after they have lost their significance as scientific terms or as concepts guiding the inventive work of a real composer.

The definition of the minor scale as the tones created by the triad 3—5—15 (and no other) leads quickly into discord with musical practice. In the scale 15—135—9—5—45—3—27—15 the dominant harmony 45—27—135 is *not* used as the *leading chord* in practice. The harmony 45—225—135 is at present generally used as the leading chord, 225 = G♯ being substituted for 27 = G.

Altho Helmholtz, first hesitating, ultimately threw the weight of his great name as a man of science into the balance of the theory which excludes every prime number except those contained in the series 1, 2, 3, 4, 5, 6, the experiential facts of music insufficiently taken care of by that theory brought about repeated attempts at introducing the number 7. Two of them, at least, are worth mentioning here. Herzogenberg, a German musician and teacher, in an article in the *Vierteljahrsschrift für Musikwissenschaft*, vol. 10, pleaded

for the 7 in minor triads. *He proposed to substitute* the triad 3—7—9 for the triad 5—3—15. Multiplying the first triad by 5 and the second by 3, we find that the triad 15—35—45 is substituted by Herzogenberg for the triad 15—9—45. We agree with Herzogenberg that a minor triad may be perceived by the listener in either way, but we cannot admit that a minor triad *in real music must* always be perceived as 15—35—45. Doubtless, if our axioms are to be relied upon, the most common way of perceiving the minor triad is that of 15—9—45 where all three affinities are of a high degree. Only in comparatively rare cases, under the influence of particular preceding or following melodic motives or chords, is the minor chord of musical practice likely to be perceived as 15—35—45. Herzogenberg, being struck by a concept thus far practically neglected, overgeneralized the new concept and gave it the appearance of being a cornerstone, when really it was only a stone filling a hole in the wall.

Another attempt at introducing the 7 was made much earlier, twelve years before the publication of Helmholtz's work, by an American organ builder, H. W. Poole. In *Silliman's Journal*, 1850, Poole reports having experimented with playing many pieces of music in a new intonation, which seemed to him to produce a superior esthetic effect. This intonation corresponds to the following definition of the diatonic scale: 1 — 9 — 5 — 21 — 3 — 27 — 15 — 1, where 21 is obviously a product of 7. We must say here of Poole what we said above of Herzogenberg: he overgeneralized his new concept. The new definition of the diatonic scale does not logically require the expulsion of the older ones. Some pieces of music because of their total structure are most likely to be perceived in one way, others in other ways. And some pieces of music because of their structure may be just as likely to be perceived in one way as in another way, so that temporary or permanent idiocrasies of hearers then alone determine whatever perception may be chosen. In either case the music is good music. For comparison in another field of esthetics think of a landscape painting which contains a child hidden among rushes so that you are uncertain if the child wears pants or a skirt, that is, if it is a boy or a girl. The painting produces an esthetic effect none the less.

Helmholtz, however, entirely failed to see the theoretical significance of Poole's work,—another proof of the fact that he had been completely captivated by Rameau's theory of the diatonic scale. Poole's interests, on the other hand, were chiefly of a technical nature and caused him to devote himself to the invention of keyboards permitting the use of his new scale in different keys rather than to research concerning the theoretical value of his scale. Thus musical theory remained practically uninfluenced by Poole's work.

While the pretense of Rameau that his *dualistic* theory of harmony (major-minor) had a *scientific* basis had been more or less *challenged* by his

contemporary, the *physicist* D'Alembert, a later *physicist*, A. von Oettingen,[31] not only *defended* the logic of Rameau so far as mere dualism is in question, but even *tried to strengthen the dualism* by a new logical derivation of the "symmetry" upon which the notion of dualism is based.

Let us at once point out, however, that the major-minor theory of Oettingen is worse than Rameau's in this respect: that it fits even fewer, indeed only an exceedingly small minority of, cases of musical experience.

Reasoning from the fact that Rameau proposed 1 — 3 — 5 as the make-up of the major chord with 1 as the chief tone, and 3 — 5 — 15 as the make-up of the minor chord with 5 as the chief tone, Oettingen very unreasonably invites the reader to *form the reciprocals of the latter three numbers*, that is, 1/3, 1/5, 1/15. Oettingen then gives these fractions a common denominator, 15, and considers then only the numerators 5, 3 and 1 (in 5/15, 3/15 and 1/15); and he invites the reader to conclude that, because 1 thus sprang from 15 and because 1 was *the chief* tone in the other (major) case, therefore the chief tone of the minor chord 3 — 5 — 15 must be the tone 15, *the father of the chief* 1. This would mean, for example, that among the tones C — A — E heard the tone E would have to be the chief tone. Oettingen comes very near rewriting all the scores of all the previously composed music in minor, for the purpose of correcting all the "mistakes" made by the composers who were ignorant of his logical discovery. Musicians, of course, have not been willing to sacrifice their quite different musical experiences to Oettingen's "logic."

Hugo Riemann[32] alone among the musicians was captivated by Oettingen's attempt at imposing the duality notion of Rameau more strongly than ever upon musical thinkers,—captivated to such a degree that he even had hallucinations; for at one time he asserted the existence of "undertones", corresponding to those reciprocals mentioned in the previous paragraph, as the acoustical counterpart of "overtones." The major scale would then be governed by overtones, the minor scale by undertones. What a beautiful "symmetry"! And not only a mysterious and speculative symmetry, but a truly acoustical symmetry, strictly scientific! Undertones versus overtones!

Unfortunately (for Riemann) acoustical science knows nothing of undertones. These hallucinatory tones would have to be mechanically *multiplex* tones in the same sense in which overtones are mechanically *partial* tones. But this would require the imagination of a sort of fourth dimension for space. A volume of mass under tension can indeed vibrate in parts of itself. This is plain mechanics. But how could it vibrate in multiples of itself? The multiples would have to lie in the fourth, the invisible dimension.

31 *Harmoniesystem in dualer Entwicklung*, 1866.

32 *Musikalische Syntaxis*, 1877.

From all this nonsense we should learn this lesson: We ought to abstain altogether from mixing up acoustics (which is nothing but a branch of mechanics) with musical theory (which concerns itself with reactions of organisms equipped with nervous tissue), notwithstanding the fact that on occasions a knowledge of acoustics can be very valuable to certain friends and helpers of the musician, that is, to the builder of musical instruments and to the physician treating larynx troubles.

The Diatonic Scale Giving Birth to the Tonal Alphabet. Whoever is even slightly familiar with musical notation and nomenclature, if he is at all inclined to ask curious questions, must have asked himself on some occasion this question: Why does the tonal alphabet not run thru the first twelve letters, *a b c d e f g h i j k l* ? Why does it in English usage begin with *c*, run in the manner of the regular alphabet thru *g* and then add *a* and *b*; and why does it in German usage begin with *c*, run thru *g* and then add *a* and *h*? And why is the need of five lacking letters filled *doubly* by such queer symbols as *d*-flat or *c*-sharp, and four other monstrous twins? Acquaintance with the chief facts of the history of this alphabet has the same advantages as acquaintance with the origins of the diatonic scale, that is, it discourages fantastic speculation and encourages a truly scientific attitude toward the problems of musical theory. The nomenclature has taken its origin from *a combination* of the experiences of musical practice *with mere accidents* of theoretical tradition and speculation rather than from systematic investigation. Historical knowledge helps to free us from the suggestiveness of names which is apt to turn our musical perception into misleading illusions, when we know for example that a "natural" note does not actually mean that it sounds natural, unartificial, to our ear; or when we know that, in German usage, "dur" and "moll" do not actually mean that a major scale *sounds hard* and a minor scale *soft*, however differently they do impress us.

There seems to be no doubt that the most common musical instrument of early Greek history was a string instrument of four strings, called tetrachord.

The Roman musicians of the early middle ages accepted the ancient tonal systems which had been developed from the tetrachord as the theoretical foundation of their art. But they did not favor the somewhat cumbersome Greek names of the tones of the various strings. Some one invented a simple method of naming the tones by using the letters of the alphabet. By an historical accident he chose as the tone system to which he applied the letters the one represented here in number symbols: 5 — 45 — 3 — 27 — 15 — 1 — 9 — 5. He did not, however, stop at the letter *g*, but continued the alphabet, using for the following tones *h*, *i*, etc. The now universal habit of calling notes which form the interval of an octave by the same name, thus avoiding the use of any but the first seven letters of the alphabet, was introduced about the year 600 by Pope Gregory.

The next step in the development of the tonal alphabet resulted from the necessity of transposition. It was frequently found that the singers had difficulty in following the instrument, the church organ, when the tune proceeded considerably towards the lower region. To overcome this difficulty, the player transposed the tune by taking each tone three steps higher on the string instrument or organ. It is easy to understand why the player should have transposed the tune just by three steps. It's the span of the Greek tetrachord.

Beginning now, e. g., with *f* instead of *c*, and ending on *f*, the intervals are 2s, 2s, 2s, s, 2s, 2s, s. This is clearly not quite the same tune as beginning with *c* and ending on *c'*, the third and fourth intervals being different in the two cases. In a string instrument this difficulty could be overcome by simply altering the tuning of the string *b*, by tuning it a semitone lower when the tune is transposed from *c* to *f*. But the tuning of the organ pipe cannot so easily be changed. Therefore a pipe was added, so that the instrument contained two tones between *a* and *c*, that is, two *b's*. These *b's* had to be distinguished both in writing and in speaking. In writing they were distinguished by making the higher *b* square and the lower *b* round. In speaking they were distinguished by calling the square *b* either "*b* quadratum" or "*b* durum", because, being square, it *looked* like a hard (Latin *durum*) thing, say a quarry stone; and by calling the round *b* either "*b* rotundum" or "*b* molle," because, being round, it *looked* like a soft (Latin *molle*) thing, say a pillow. The terms *durum* and *molle* (English: *hard* and *soft*) had no reference whatever to the esthetic effect of the *sound*. They could not have such a reference since *the tune was exactly the same tune in both cases*, only transposed.

The distinction of the two *b's* has become most important for modern musical notation and phraseology. In Germany at a very early period[33] the letter *h* was substituted for the square *b*, obviously for these two reasons: The square *b looks* somewhat like the letter *h*; and the last letter of the alphabet used in the older systems being *g*, suggests the use of the next following letter, *h*, of the alphabet to replace one of the *b's*. The outcome of this development is that at present the tone which is called *b* in English is called *h* in German, and the tone which is called *b* in German is called *b-flat* in English. Let no one accept, however, the idea that *h* is an abbreviation of *hard* (German *hart*), and that it was used to indicate the hardness of the tune. The letter *h* has no reference whatever to hardness of the esthetic effect, since the tune, as we saw, was the same, merely transposed. Besides, the medieval musicians who first used the letter *h* could not use it as an abbreviation of the German

33 On the key-boards of the fifteenth century described in the second volume of the *Syntagma* by Michael Praetorius, the corresponding keys were marked with the letters *b* and *h*, as we mark the keys of our typewriters.

word *hart* because even in Germany they did not employ in their profession the German language, but spoke and wrote in Latin.

During the following centuries the use of the musical alphabet, and of the round *b* and the square *b* within it, underwent further remarkable changes. When the great significance of the *c* in the diatonic scale was clearly understood, when it was generally recognized that in the system of tones *c* was the King, so to speak, of the whole group, the *c* was used as the first and the last letter of the musical alphabet, *c, d, e, f, g, a, h, c'* in German.

The first transposition was, as we have seen, a transposition by a fourth upward. When the musicians began to use additional transpositions, they found it necessary to change further tones of the original scale and to invent symbols indicating such changes. Starting from the round *b* instead of from *f*, it was necessary to change the *e* into a tone *a semitone lower*. To indicate this change by a symbol, they generalized the meaning of the round *b* and simply added the round *b* to the *e* or to any letter as a sign that the tone was a semitone lower. However, it was found desirable to distinguish the generalized meaning of the round *b* from its original, individual meaning. Therefore they used as a general symbol for lowering a tone the letter *b* pointed at the bottom, ♭.

The further development differed somewhat in different countries. In England the distinction in writing between a square and a round *b* was abolished. The higher one was written and pronounced simply *b*, and the lower one was written *b*♭ and pronounced *b flat*. The other letters, when lowered, were written *e*♭, *a*♭, *d*♭, and so on, and pronounced *e-flat*, *a-flat*, *d-flat*. French musicians instead of saying "flat" say "bémol". And the Spanish say the same, "bemol." In Germany *b* and *h* were retained and are pronounced in the ordinary way. But *e*♭, *a*♭, *d*♭, and so on, are pronounced *es*, *as*, *des*, *ges*, *ces*.

In the seventeenth century it was found desirable to have also a symbol which called attention to the fact that a tone which in a former part of the piece of music had been lowered, was now no longer to be lowered. For this purpose the musicians generalized the meaning of the square *b*; and, in order to distinguish it from the square *b* in its original, individual meaning, they chose another form of the square *b* to be used only as a writing symbol in this generalized meaning. Thus originated the modern symbol ♮, which is pronounced in English *natural*—not because the tone is more closely related to Nature, but because it is useful to call special attention to the fact that there is no longer any alteration of the tone. In German the symbol ♮ is called *Wiederherstellungszeichen* but is not pronounced at all.

When the medieval musicians learned to use transposition more freely they began to transpose downwards as well as upwards by one or more tetrachords, that is, in modern phraseology, intervals of a fourth. Passing down a fourth from *c* they reached *g*. When they now played the tune made up of

the intervals 2s, 2s, s, 2s, 2s, 2s, s beginning with *g*, instead of *a*, they found it necessary to alter the *f*, to make it *a semitone higher*. In order to indicate this in writing a symbol was needed. It is but natural that, having previously generalized the meaning of the round *b*, they now generalized the meaning of the square *b* and used it to indicate that the tone to which they added it was higher. But it was necessary to distinguish the square *b* in this sense from the other square *b* then already used for any unaltered tone of the scale. They therefore invented a modified way of writing the square *b*, choosing the form of a cross of parallels, ♯. This is now pronounced in English, *sharp*, in French *dièse*, in Spanish *sostenido*. In German f♯, c♯, g♯, and so on, are pronounced *fis*, *cis*, *gis*, *dis*, *aïs*. We thus actually have nowadays four forms of writing the letter *b* in music. Few people who have taken music lessons know that these four forms are all nothing but *b's*.

The terms *dur* and *moll* still survive in the German language. They are usable for translating the words *major* and *minor* in the terminology of the English musicians into the German language.

A Practical Conclusion Suggested by the Author. Let us end this history with a brief question and a brief answer.

Question—What would be most helpful in music today to the composer, to the performer, and to the theorizer?

Answer—If we could bring it about, that the diatonic scale be "spurlos versenkt" and that, instead of busying himself with *absorbing* "a" scale (that is, "the" diatonic scale of the past), the music pupil devote his time to *the invention of scales* for his own fun and for entertaining others with music built of the elements of such scales of his invention.

APPENDIX 5. A BRIEF HISTORY OF THE THEORY OF CHORDS

We have agreed not to overemphasize for musical theory the practical effects of "freezing" a melody into a harmony.

Nevertheless we admit that certain discoveries concerning the variety of the esthetic effects of grouping such and such tones may, *historically*, very well have been made first when musicians became accustomed to systematic harmony, and never before, i. e. never in mere successive grouping. We should expect from our axioms that medieval music teachers, when beginning *systematically* to teach the making of *harmonized* music, would before their students give expression to the experience that *various intervals differ greatly in their esthetic effects*. These effects, in so far as they (to us nowadays seem to) *depend on biochemical affinities* within the nervous tissue of the body, are naturally somewhat more pronounced when the neurological functions occur at the very same time than when they occur at different times, however temporally close the *succession* may be.

Other esthetic effects of tone combinations *depend on secondary phenomena* such as roughness of the sound, beats, the unintended but unavoidable *induced tones*[34] with their affinities to the intended tones, etc. These secondary phenomena being *absent in melody*, their (esthetic?) effects can be said to be present *only in harmony*. It is thus but natural that musicians when first beginning for scholastic purposes to describe music with respect to the esthetic inequalities, should have chosen harmony rather than melody for the discussion with their students.

Further, each melody, like a spoken sentence, consists as a rule of many, many phrases; and each phrase consists but *rarely of two* tones only. A melodic phrase, therefore, because of its complexity does not offer any ready suggestion *as to a method* of analyzing it psychologically for the purpose of a scientific description by reference to *pairs of tones*. A harmony much more so, especially if so simple as the earliest harmonies of the tenth century, which consisted of two tones only.

In order to describe the esthetic differences of different harmonies of two tones each, the medieval musicians naturally classified the harmonies under two headings as consonant harmonies, that is, very pleasing harmonies, and dissonant harmonies, that is, less pleasing or even relatively unpleasant harmonies. What else could one expect of them for a start?

Under each heading, "consonant" or "dissonant", they enumerated the harmonies by naming them in accordance with *the number of keys* of the diatonic scale *needed to cover the interval*. Thus we find the music teachers of the Middle Ages (no matter how absurd, confusing, ambiguous this appears immediately to the acoustician of the twentieth century) discussing quite naively whether "the" second or "the" third or "the" fourth or "the" fifth or

34 *The Journal of General Psychology* (April, 1928) pp. 254-255.

"the" sixth or "the" seventh or "the" eighth (there happens to be only one) or "the" ninth or "the" tenth, and so forth, *is a consonance or a dissonance*,— without noticing that "the" combination was a plurality of combinations, that there were *two* different seconds, and so forth.

The enormous force with which in human society traditional conceptions hold their places against reason and experience to the contrary, can be seen illustrated by the survival value of these two concepts, in spite of the fact that it was soon vaguely felt that the terms "consonance" and "dissonance" were ambiguous, open to various interpretations. The terms have survived. And few music pupils in the twentieth century escape being taught the use of these ill chosen terms as if they were highly valuable.

Many modern psychologists, unfortunately, have fortified the life of these ambiguous terms by reviving a secondary definition of "consonance" and "dissonance" when musicians were almost ready to discard the terms—the definition by reference to secondary phenomena such as roughness and smoothness in simultaneous hearing. Modern music only in the art of orchestration and only to a slight extent takes account of roughness and smoothness; otherwise it has learned to disregard and has succeeded in training the hearer to disregard secondary phenomena almost entirely. But psychologists, in moments of idleness, fished the drowned corpse out of the water and revived it with oxygen, not seeing how much harm they did to science.[35]

The primary definition of consonance and dissonance, let us remember, should always rest purely on the degree of affinity of two tones in tunes.

In the Middle Ages at first only three intervals were called consonant by the harmony teachers, the octave, the fifth and the fourth. Larger intervals, as the tenth and twelfth, were of course scarcely ever used at the earliest period when harmonies consisted of two tones only.

Nothing could be more wrong, more unjust, than to conclude that the counting of the thirds as dissonances was the result of careless observation by the medieval harmony teachers. Nobody called their attention to tone affinity in the abstract. So they described the *total* esthetic effect, including such secondary phenomena as roughness. Observation in the laboratory of acoustics will easily convince anybody that disturbing *secondary* phenomena increase as we compare successively the octave, the fifths, the fourth, the thirds and sixths. In drawing the line after the fourth and before the third, however, the medieval musicians were surely influenced by the fact that in pure affinity the fourth and the fifth are peers.

Let us see why with the advancing musical experience this classification of harmonies into consonances and dissonances, accords and discords, proved unsatisfactory and called for a modification.

The Theory of Chords under the Influence of the Organum. The earliest

35 For example, C. E. Seashore in his tests of "musical talent." Compare footnote 36.

attempt at harmonizing music, beginning probably in the ninth century, consisted in adding to a given melody a second voice singing each tone either a fourth or a fifth higher, and later a third voice singing in the interval of an octave from the lowest tone. Everybody knows that this kind of polyphony, called organum, is described by Hucbald, a writer of the tenth century; but it was probably known long before him.

Modern writers usually cannot find contemptuous epithets strong enough to apply to such a "harmonization", but this way of judging it reveals a deficiency of both historical and psychological insight. The traveler in a sparsely settled country, accustomed to the dinner served in the dining room of a city hotel, may swear at the meal which he buys at a little crossroad store; but his emotion does not prove that this meal is not truly delicious in comparison with the half-raw food available to primitive man.

Naturally, whoever has heard and enjoyed the music of later centuries, soon despises the early kind. But those who throw contemptuous epithets on the earliest attempts at harmony would probably have been slow in inventing a better kind themselves if they had not found a better kind already in existence. It sounds so simple to say: Why did not the musicians of the tenth century run their fingers over the keyboard, comparing the sound of successive fifths or fourths with that of major or minor thirds? But the clumsy keys, struck with the fist, of the wretchedly voiced and tuned organs of the time did not invite to such experiments as the ivory keys of a piano do. Experiments, even the simplest, are rarely made unless someone or something suggests the experiment; for an illustration of this assertion remember the legendary story of the egg of Columbus.

Besides, the *average* musician has always distrusted and will always distrust his ear *unless he feels supported by a theory* derived from previous experience of himself or—usually—of his teachers. Making allowance for some exceptions, there are only two classes of musicians who are likely to make inventions by experiment, (1) geniuses possessed of great initiative, of whom there are but a few in a century, and (2) ordinary, but somewhat impulsive, mortals who in consequence of life's fortunes have remained untaught and thus are neither guided nor hampered by theories. The average music pupil of the tenth century *was taught* that the octave, the fifth, and the fourth produced harmonies superior to all others, consonances instead of dissonances. He found nothing in experience, i. e., in the total esthetic impression made by different intervals, to the contrary. Why not? Because his teachers, developing *the first* theoretic description of harmony, described the simple, not the complex. They described to the pupil separate harmonies, *not harmony successions*. The pupil could not suddenly conceive the idea of *describing the successions and of experimenting to this effect.* Modern music pupils do not surprise their teachers by such originality either. It is only in successions of harmonies that the interval of the third reveals its beauty.

The Theory of Chords under the Influence of the Discant. From all we know about the harmonic music of the eleventh and the following centuries, there seems to be no doubt that the progress was due largely to the impulsiveness of musicians of the second class just mentioned, ordinary singers who, not being either guided or hampered by theory, amused themselves by singing simultaneously two entirely independent melodies. We find amusements of a similar kind with children in our own time. Success was made possible by the fact that medieval melodies did not have a rhythmic structure as definite as that of modern music, so that in time relations two melodies could easily be adjusted to each other in such a manner that a certain note of one melody could fall on a certain note of the other melody, according to the intentions of the singers. Since the stately, slowly moving chant of the church seemed to need *variety* especially, the singers tried to give it this variety by combining with it a lively secular tune, sung higher because it is more difficult to sing a lively tune in a very low voice.

How naive, uninfluenced by any theory of artistic construction, the procedure of these child-like singers was, becomes apparent from the fact that they did *not* even change the *words:* the singer who sang the secular melody used the secular words, and the singer who sang the sacred melody used the sacred words. This habit of welding secular and sacred melodies with the text belonging to each into a single piece became quite universal for centuries.

The *whole* piece was called a discant. *Gradually "discant" came to mean only the upper, more lively melody*; and the lower melody was then called either *tenor*, meaning literally a 'holder', a support, or *bordone* (Italian), *bourdon* (French), meaning again a support, but by a figure of speech, since *bordone* literally means a staff, especially a pilgrim's staff. Later these same terms assumed various other meanings under which they are still used in musical practice. For example, we find bourdon as the name of an organ stop.

The *discant* which we have just briefly described could not help leading gradually to a clearer comprehension of *the esthetic significance* in musical practice of harmonies *other* than the octave, the fifth, and the fourth. The result was that the music teachers felt compelled to make room for the new experiences in their theories. The general *trend of the theory* (almost up to the present time) became a two-fold one: (1) The number of "consonances" steadily increased, the thirds were admitted, the sixths, the minor seventh, the ninth, and thus of course also the second, finally even the tritone and the major seventh: *The flexible concept of a "chord" took the place of the rigid concept of a consonance.* (2) The fourth was soon expelled by the theorist from its place of honor among the consonances and was relegated to the dissonances, only in order to be reluctantly accepted again as a chord interval, when the concept of consonances and dissonances was replaced by the concept of chords together with the notion that what the theorist did not want to call chords

should be called *accidental* combinations of simultaneously sounding tones or chords with *accidental notes* mixed in. All this seems strange and confusing to the unbiased man of science *who knows no accidents.*

The at first reckless and bold attempts of theoretically untutored singers at "discanting", becoming gradually, with increasing experience, more successful from an esthetic viewpoint, could not fail to draw the attention of the music teachers, the theorists. What could they do but *admit that there were other consonances* than the octave, the fifth, and the fourth, since other intervals occurring between simultaneous tones of the discant had already *proved to make good music?* But they would naturally take these tone groups out of the complex in which they heard them, and study them separately, carefully comparing the total esthetic effect of each of the newly invented duads, with the total esthetic effect of the octave, the fifth, and the fourth. The experience in the discant had proved that the intervals of three, four, eight, and nine semitones, the thirds and the sixths, had to be saved from being treated as the weeds of the musical garden.

The music teachers, having only the two terms consonance and dissonance for the description of the facts, proceeded in the same way as the human mind, save rare exceptions, always proceeds. They kept these terms, but slightly modified them, just enough to express distinctions which they observed.

Now a comparison of the *total* esthetic effect of the various duads showed that it was *not* advisable to classify the thirds and sixths *together with* the intervals hitherto called consonances. We easily understand this. The thirds and sixths have a lesser degree of affinity and a greater amount of disturbing secondary phenomena than the fourth and the fifth. Two ways of modifying the traditional terminology suggested themselves. A distinction could be made either between perfect and imperfect consonances *or* between perfect and imperfect dissonances; and the thirds and sixths could be classed either as imperfect consonances or as imperfect dissonances.

Thus we find at the end of the twelfth century (for example, in the doctrine of Franco of Cologne), *after three hundred years of experience with harmony,* the theory of harmony using the following classification:

1. Perfect Consonances: Unison and octave.
2. Medium Consonances: Fourth and fifth.
3. Imperfect Consonances: Thirds.
4. Imperfect Dissonances: Sixths.
5. Perfect Dissonances: Seconds, tritone, and sevenths.

We can at present comprehend that this theory was not destined to become a permanent theory of scientific value. The principle of classification was not sufficiently abstract. The classification was based on *the judgment* of

the concrete total esthetic impression made upon the hearer[36] by a duad supposedly separated from all connections—an impression which is the result, not of a single condition, but of a complexity of conditions. The medieval musicians did not succeed in developing useful abstract concepts. Rameau only made an important beginning in this direction five hundred years later, although the dispute concerning the proper classification of the fourth, which we shall at once discuss, might have led to a much earlier beginning of this advancement.

"All's well that ends well" is applicable to art no less than to affairs. If perceptual unity is an important esthetic factor in music, it must be especially important *at the end of the piece*—as the theorist will reason. The last harmony must represent a consonance rather than a dissonance, a perfect rather than an imperfect consonance—thus the music teachers would reason. But the experience of the hearer did not agree in this case with the theory of the teacher, for the fourth, although classified theoretically with the fifth, did not make as good an ending as the fifth,—indeed, appeared to be quite impossible. We can now easily understand this. Both intervals are represented by the same affinity, 1—3, but *the direction* of the effect is different. *Be sure to remember* what was said below problem 42 about the effect of a pure power of 2. Further remember that in the early discant the lower melody was as a rule —in church music universally—the chief melody, the 'tenor', the 'chant'. The lively secular tune was added to the stately chant as an upper voice,—naturally thus, as we have seen. The sacred words belonged to the lower voice; the upper voice was added merely as an ornamentation.

Then, of course, *the fourth was not adapted to be used in the final harmony*. The effect of the pure power of 2 would have led the hearer's attention away from the last tone of the chant to the last tone of the ornamentation, the discant. *While we find no difficulty in understanding this, to the medieval theorists it was a puzzle, owing to their not possessing our concepts, and their attempting to describe everything in terms of consonance and dissonance.* This restriction of the use of the fourth in a final harmony, the necessity of having

36 The so-called "consonance" test among Seashore's tests of musical talent unfortunately adopts this medieval procedure on the authority of Carl Stumpf's "Tonpsychologie". Thirty years ago I pointed out in three publications that "consonance" (also called "fusion" by Stumpf) is not a term corresponding to a single psychological concept, that it is not capable of being defined unambiguously: "Ueber Tonverschmelzung und die Theorie der Konsonanz", "Nachtrag zu dieser Abhandlung", "Ueber Beurteilung zusammengesetzter Klänge". *Zeitschrift fur Psycholgie und Physiologie der Sinnesorgane* XVII, pp. 401-421, 1898; XVIII, pp. 274-293, 1898; XX, pp. 13-33, 1899. It took thirty years before any other psychologist joined me in assuming an attitude as critical as mine toward the term "consonance" and similar terms in simultaneous and successive tones: Compare C. P. Heinlein's critical articles in the *J. Exper. Psychol.*, 1925, pp. 408-433, and the *J. Genetic Psychol.*, 1928, pp. 45-61. Until Heinlein's criticism the medieval notion, taken up by Stumpf, was uncritically copied in virtually all psychology texts, monographs, and smaller publications in psychological magazines.

it succeeded by another harmony, did not agree with the adopted classification of the duads as consonances and dissonances, and yet this classification otherwise seemed to fulfil all requirements.

The music teachers for centuries could not agree as to *the rank of the fourth*, *some* preferring to call it a *dissonance*, *others* preferring to call it a *consonance*, *until* a certain amount of clearness was brought into the matter by *Rameau*, but not enough to satisfy all,—and so we find the absurd struggle about the "consonance or dissonance" of the fourth going on still at the end of the nineteenth century.

The Theory of Chords under the Influence of the Double Discant. When the attempt at sounding two voices simultaneously had met with a considerable degree of success in the discant, the endeavor was natural to try the combination of three voices, called the double discant. But this was obviously too complex and difficult a task for the untutored singer. Success here came from the side of those musicians who permitted themselves to be *guided by rules*, *by theory*, and who were anxious to develop the theory in order to be able to teach their pupils to do what they had themselves succeeded in doing. The *only kind* of three-part music known thus far was the so-called *organum*, in which to each tone of the chant were added two higher tones, the fifth or the fourth, and the octave—with a few early exceptions like John of Forsete, the composer of "Sumer Is Icumen In". But theorists then preferred to measure each interval *from the next lowest tone* rather than from the lowest tone of the harmony. They would describe C — F — c, not as the fourth and the octave, but as a fourth and a fifth; and C — G — c, not as the fifth and the octave, but as a fifth (C — G) and a fourth (G — c). Now the discovery had been made that other intervals, too, made agreeable music, that indeed they added a charm not otherwise attainable. The theorists, inclined to be as conservative as possible, would naturally reason that, *if other intervals had* to be introduced, those should be selected which are the least dissonant (see Franco's table above). So they chose the imperfect consonances, the intervals of four and three semitones, the thirds, as we call them now, *the ditone and the semiditone as they were then called.*

The question to be decided next was, *which of the intervals of the three-part organum was to be replaced* by either of the thirds. Now, *if* they had substituted *a third* (either three or four semitones) *for the fourth*, the sum of the seven semitones of the fifth and these three or four would have produced between the extreme tones of the harmony an interval of either ten or eleven semitones, a seventh, a 'perfect dissonance'. But by substituting either of the thirds for the fifth of the organum, the *extreme interval* of the new harmony, a fourth plus a third, would be made to cover either one of the sixths, as we now say, (a hexachord, according to the medieval language) an imperfect dissonance, theoretically superior to the perfect dissonance of a seventh. Of

course, under these conditions, the fifth, not the fourth, had to make room for the third.

So the new harmony was to consist of a third plus a fourth. Only the final (and usually also the initial) harmony of the piece had to be more consonant, so the musicians argued. Therefore in the *last* (*and usually the first*) harmony the third was again replaced by the original fifth so that the ending (and beginning) of the piece was made in the manner of the old three-part organum, by a combination of a fourth and a fifth.

However, a very important question was still unsettled. Was the *fourth to be the upper* or the *lower interval*? This question could be answered *on the basis of the experience* above mentioned: The fourth had been found to be an impossible interval in the final harmony—as we say, because the effect of the pure power of 2 leads away the hearer's attention from the final tone of the chant to the tone of the discant. This could be avoided, whenever a *double discant* was added, by making by general rule the *third* the *lower*, the fourth the upper interval. When now in the final harmony *the fifth took the place of the third*, the hearer's attention was led toward the tone of the chant (the power of 2), not away from it.

One other rule had to be added in order to establish a definite set of rules by which the music pupils could be taught to let themselves be guided. When the tone of the chant was D, the third above D was F, and the fourth above F was B, in the diatonic scale. But the interval, F—B, was *not exactly a fourth* equal to five semitones, but a tritone, equal to six semitones, a perfect dissonance. Without any sufficient attempt at experimenting, the music teachers ruled that no such impurity was here allowed and that the interval F — B, not being a true fourth, had to be avoided. This could be done in two ways. *Either* by using for the accompaniment of the tone D of the chant the same harmony which was prescribed in case the chant was of a mode in which D was the final tone, namely the harmony D — A — d, made up of a fifth and a fourth; *or* by avoiding the tone D in the chant.

A remarkably interesting (compared with the old organum) kind of three part music had thus been discovered, the composition of which could easily be taught, since a complete description of its construction was contained in the simple rule: let the upper interval be a fourth, the lower one a third, except in the final and preferably also in the initial harmony where the lower interval must be a fifth; and avoid the tritone harmonies, D ↗ F ↗ B and B ↗ F ↗ B (either F ↗ B or B ↗ F was called a "tritone") using in their stead D ↗ A ↗ D and B ↗ D ↗ G.

This kind of polyphony was called 'falso bordone' (faux bourdon). The meaning of the term is readily understood. We saw that the name *bordone* was originally given to a *chant* to distinguish it from the ornamental voice, which was a fifth or a fourth higher. Now, in the newly discovered polyphony,

the *true chant* was neither a fifth nor a fourth, but a sixth below the higher voice. It was not, therefore, a bordone in the traditional sense. The middle voice, on the other hand, while being a fourth below the highest voice, was not the lowest voice and therefore not entitled to the traditional name of bordone.

In *whatever sense* one would speak of a bordone, it *always was a false sense.* These falsi bordoni or faux bourdons were popular from the fourteenth to the sixteenth century. The music pupil who had learned the rule of faburden knew without having to be a genius how to make music which was no less good than that of other pupils; and this is the very thing the average pupil strives for.

The Counterpoint with its Prohibitions of Certain Parallels. Meanwhile musical instinct brought about many new ways of combining intervals into music, not only of three, but of four, five, and more voices. All the twelve intervals possible gradually became common features of any music. The theory invented by the music teachers in order to describe and distinguish what seemed to them good music and what seemed bad music was now called the counterpoint. Very simple forms, especially also in respect to rhythm, of combining voices are those represented by the organum and the faux bourdon, where theoretically one tone of one voice was considered in relation to one tone of another voice. In writing such simple music one sets note (point) against note. This is the origin of the word *counterpoint.* This word is applicable, of course, also in case the movement of the two voices *is not parallel.*

The much greater variety possible in a succession of harmonies if the movement of the voices does not have to be parallel, but may also be contrary or at least oblique, led unfortunately to an overestimation of contrary motion as such, without any reference to the significance for affinitive variety and unity, and even to *the prohibition* of parallel (and oblique) motion at least for certain intervals. These prohibitions have of course nothing to justify them except that they help to confine the composer within the bounds of a trivial sweetness.

When only one note of one voice was placed against one note of another voice, this was called *simple* counterpoint; when two or more notes of one voice were placed against one note of another voice, the term *florid* counterpoint was used. Innumerable rules for the succession of harmonies were invented, bringing some, but no perfect, clearness into the theory of music.

These rules were often contradictory, chiefly because they were based on the consideration of the single interval.

Zarlino points out that (1) a minor third, dissonance, put under a fourth (5 ↗ 3 ↗ 1) sounds better than (2) a major third, consonance, put under a fourth (3 ↗ 15 ↗ 5); and that (3) a major third, consonance, put over a fourth (3 ↗ 1 ↗ 5) sounds better than (4) a minor third, dissonance, put over a fourth (15 ↗ 5 ↗ 3). This seems a puzzle to him. One must not think that he

could have solved it by taking into account the interval of the extreme voices, which is a minor sixth (5 ↗ 1) and a major sixth (3 ↗ 5) in his two more agreeable cases, and likewise a major sixth (3 ↗ 5) and a minor sixth (5 ↗ 1) in his two less agreeable cases. This could not make the matter clearer. The major sixth would sometimes have to be a consonance and sometimes to be a dissonance? And the minor sixth also would sometimes have to be a consonance and sometimes to be a dissonance? Horrid logic! Having no other abstract fundamental concepts than that of the contrast between consonance and dissonance, the theorists could not solve this puzzle of the sixths any more than that of the fourths. Being themselves a puzzle, the sixths could not contribute to a solution of the puzzle of the location of the thirds within themselves. That the cases (1) and (3) are identical in containing exactly the same affinities 1 — 3, 1 — 5, and 3 — 5 with the directing effect *centered* on 1, and that the less good cases (2) and (4) are identical in containing exactly the same affinities with the directing effect *split*, Zarlino could not comprehend because he lacked the abstract conceptions here requisite. A really great step in advance was to come for the theory of harmony only in the eighteenth century with Rameau.

Rameau and beyond Rameau. Rameau brought a certain clearness into the theory of chords by giving each tone *an absolute name* (fundamental third, fifth, etc.) in the chord without any reference to the actual intervals. This reference to the actual intervals, he discovered, could be avoided by using the concept of the inversion of chords which treats the tones like soldiers who may be ordered by the captain to take various positions in the battle formation.

The limits of this clarifying influence of Rameau we recognized when we studied his "theorem". It is only by substituting *number symbols* for such terms as Rameau's "fundamental, third, fifth, seventh, etc., natural, diminished, augmented" that we can free the theory from artificial fetters. Arithmetic is an artificial language of wonderful possibilities, simple and yet capable of infinite extension. Why not use arithmetic rather than the cumbersome terms of colloquial language?

The number symbol has the advantage over all other terms that it is both absolute and relative. The whole struggle in the thousand-year-old history of the theory of chords, as we have seen, has been due to the double necessity of giving each tone an *individual* name and of giving it also a name *relative* to any and all other tones in the chord or melodic phrase or piece of music. Only number symbols can simply and directly and without modifying epithets fulfill this double condition. All other names force us to use queer modifiers like "augmented, sharpened," or what not, of little definiteness.

A number is always absolute, individual, in being distinct from all other numbers. And it is always relative because it permits and invites the formation

of a ratio. And there is no lower nor upper limit to the quantity of terms which may enter a ratio,—two, three, four, five, any multitude. The crazy concept of a "triad" as the only legitimate tone family, outlawing all smaller and larger families as being of illegal size, is safely avoided. For examples compare in the body of our text our numerous "scales" varying in size from two up.

Ratios, when *reduced* to their lowest terms or *translated* into "spans" are quickly comparable with other ratios without any possibility of ambiguity. No other terms are safe from ambiguity.

APPENDIX 6. CHINESE SCALE THEORIES[37]

By K. C. Hsiao

European writers are wont to style China "The Mystery of the Orient." When matters are not understood, they become a mystery. The correct understanding is in this case no doubt a very difficult task. Few occidental observers are versed in the Chinese language or acquainted with the intimate life of China. And with regard to the matter of music, China is perhaps at the height of her mystery. Chinese music is a mystery even to most of the Chinese themselves.

Music has become a sadly neglected phase of Chinese civilization, altho it existed in a relatively well developed form as early as 2852 B. C. More than two thousand years before Christ it received the ardent patronage of great emperors and high court officials. The educational and cultural values of the art of music were enthusiastically defended by the great teacher Confucius (Kung-futsze, 551-478 B. C.).

Music in China not only has become a thing of neglect, but even, in certain instances, a thing of contempt and aversion to the "cultured" person. Music is understood by practically nobody, because it is not worth the trouble of understanding. When a western student wishes to know something about the music of China, not knowing the language he naturally goes to some of his Chinese friends and asks them about it. But what these persons can tell him is likely to be something the correctness and informative value of which is rather doubtful.

37 This brief account of the history of Chinese music was written at my request, to furnish me much desired information, by a Chinese gentleman who combines a profound knowledge of Chinese scholarship and life with an intimate knowledge of western science and western practices. Maybe the reader will be struck, as the present writer was, with the fact that the histories of Chinese and of western music have in their essentials far more points of similarity than of difference.

It is interesting to note that (1) the common notion of Chinese music being more intricate than western music is a delusion, and (2) that there, in China, practice was influenced by theory and, inversely, theory by technical and social practice just as we observe this in the western world. The main difference seems to be that Chinese music started 2000 years sooner, and that even before the Christian era it suffered an arrest of development. Chinese music, it appears, stopped at a period of growth when it had reached a stage of development somewhat comparable to the worst follies of the European discant (a forerunner of modern jazz) before the Head of the Church sobered European music, which was then fortunately largely under his jurisdiction, by forcing it out of the craze of the discant and into a rational form of growth.

The following details are striking: Chinese theory early invented the Pythagorean scale and the cycle of fifths. Chinese practice early invented the pentatonic scale. Chinese theory early introduced *metaphysical notions* (e. g., the mystical influence of the twelve months of the year) *which are no worse* than the metaphysical notions of the westerner, our famous Zarlino (who believed in the music-limiting influence of the "six" dimensions of space). "Debussy's scale" was invented by the Chinese thousands of years before Debussy lived if we are willing to regard the male and the female groups of six each (although they

The difficulty can not be overcome by a visit to China and actual observation of musical performances there. During a long history Chinese musical practice has deviated far from the original music theory of ancient times. So it seems; but it is questionable whether it has not deviated in non-essentials far more than in essentials.

When we read a few of the histories which Europeans have written of Chinese music or of Chinese theories of music, we shall find that, while these writers in their accounts of the music system of China agree in one or two fundamental notions, they differ widely in certain important details. And that is not to be wondered at. The difficulties surrounding the study of the Chinese music system suggested above are almost insurmountable for a European who has to be contented with a second-hand knowledge of the subject; and even if he could read Chinese, the literary style in which the ancient Chinese wrote is so indefinite, metaphorical, and "metaphysical" that their music system appears already mysterious and puzzling to the Chinese reader, not to mention the helpless western friend who does not have the ability to read the language.

Nevertheless, the music system of the Chinese is in fact simple and without any considerable complications. It is built upon certain principles which are just as reasonable and in conformity with the fundamentals of esthetics as are the almost identical principles underlying the western system. Complication and mystery accidentally and unfortunately result from the over-enthusiasm of the ancient Chinese writers, from a neglectful attitude of the average modern Chinese belonging to the educated classes, and from the incorrect interpretation of information gathered, under the disadvantages already pointed out,by western writers.

In the following paragraphs the present writer will simply outline and explain the original Chinese music system as correctly as he can make it out from the writings of certain ancient philosophers and historiographers; and

are not a true scale, as properly defined) as *an analogy*—and why not? For Debussy always transgresses his six notes, and therefore does not really present music in a true six note scale; he only philosophizes in that six note scale, but does not make music thus. The appeal of certain Chinese theorizers to "five" as a *sacred* number is obviously metaphysical, but no more metaphysical than Rameau's appeal to "symmetry" as a *sacred* principle establishing musical modality and thus limiting music to "two modes" almost down to the present day. The sixty tone scale of Ching-fang is like one egg compared with another when placed side by side with the music system of our "revolutionary" (the epithet which he gives to himself) musician, Julian Carrillo, who mysteriously calls his system "el sonido trece," that is, the system of the thirteenth tone, justifying the name thus: when sixty is made out of twelve by adding, the first thing added cannot help being the thirteenth. Last, not least, in allowing a comparison,—while the Chinese introduction of quartertones (as a commercial trick of the trade, of the shop keeper) and the western and recent introduction of quartertones (for the purpose of doing something revolutionary in "glissando modulation") are not directly analogous, they are indirectly analogous in the total lack of respect for the singableness of music. (M. F. M.)

he will discuss the transitions from the original music to that which exists in actual practice. If the reader is disappointed by the simplicity of the subject thus presented, and inclined to blame the writer for destroying one of the mysteries of the Orient, the writer wishes to excuse himself by saying that this is not his fault, for the Chinese music happens to be thus simple, and there happens to be no mystery in it—nor, for that matter, in any other things Chinese.

I. The Original Chinese Music System: The Twelve Tubes of Lin Lun and the Pentatonic Scale

Music was known to the Chinese, as aforesaid, as early as 2852 B. C., in the reign of the Emperor Fu-shi. But it was not until in the reign of Huang-ti (2696 B. C.) that a definite music system was invented by the court musician Lin Lun, who made twelve bamboo tubes of certain definite lengths produce twelve notes in a chromatic series with a total span equivalent to that of an octave. This was the foundation of the Chinese music system, which later underwent only slight modifications.

The earliest Chinese musical theorist employed the principle of progression by the ratio 2:3, as Pythagoras did in Greece, although in somewhat different ways. How Lin Lun actually arrived at the twelve-note system is not actually known; the account of his procedure given by later writers seems to be questionable. It was said that he took a piece of bamboo approximately 2.9 cm. in inner circumference and 27 cm. in length and produced on it the first note of the whole series. Another piece of bamboo was cut at a length exactly two-thirds of that of the first (same circumference), that is, 18 cm., producing a note a fifth higher than the first. The first was named huang-tsung, the "key-note" of the series, and the fifth above it, ling-tsung. As the whole series must not have a span beyond the octave, owing to the limitations of primitive mechanics and materials of construction, the fifth of ling-tsung must be produced, not above it, but in the lower octave. Hence instead of making the new tube two-thirds of the length of the ling-tsung tube, he is said to have made it four-thirds of it, namely, 24 cm. in length, producing the note ta-tsou. Thus ta-tsou is five semitones lower than ling-tsung and two semitones higher than huang-tsung. The procedure was repeated from ta-tsou until the whole series of twelve tubes was completed.

The following are the names and approximate measurements in centimeters of the twelve tubes in the Lin Lun system:—

1.	Huang-tsung	27.00 cm.	(male)	— c.
2.	Ta-lui	25.28 cm.	(female)	— cis
3.	Ta-tsou	24.00 cm.	(male)	— d.
4.	Cha-tsung	22.47 cm	(female)	— dis.
5.	Ku-shien	21.33 cm.	(male)	— e.

6.	Chung-lui	19.97	cm.	(female)	— f.
7.	Sui-ping	18.96	cm.	(male)	— fis.
8.	Ling-tsung	18.00	cm.	(female)	— g.
9.	Yih-cheh	16.99	cm.	(male)	— gis.
10.	Nan-lui	16.00	cm.	(female)	— a.
11.	Wu-yih	14.98	cm.	(male)	— ais.
12.	Ying-tsung	14.22	cm.	(female)	— b.

It is clear from the above table that the twelve notes produced by the twelve tubes form a chromatic scale within the compass of an octave. The last column of the table places the system before our eyes in a manner which makes it easier for the western musician to think of the interrelations of the twelve tones than it would be if he had only the Chinese names before him. But no absolute pitch is intended to be indicated by the last column. For example, the vibrational rate of the note huang-tsung will only by rarest accident be found to be identical with that of 'c' in whatever European standards of pitch. However, the intervals between any two of the twelve notes are theoretically divided equally as in the European "equally tempered" scale.

But unfortunately the Chinese were too fond of mystical analogies which they applied freely to their theories of music. This can not be said of the creative musician Lin Lun. He did not give any "metaphysical" reason for the construction of a music system of twelve equal semitones. There is good reason to believe that he fully grasped and understood the fundamentals of the science and psychology of the musical data then available. The musicians after him, however, invariably drew minute and ridiculous triple analogies between music and astronomy and human relationship. The twelve notes of the series were made to represent the twelve months of the Chinese calendar by definite succession, and a gender was assigned to every one of them, dividing them into two groups of six notes each, one masculine and one feminine. A collective name was given to the masculine group, "lu", and the feminine, "lui".

But it must be remembered that the twelve notes of the Lin Lun system did not form a "scale." The twelve tubes were really pitch pipes rather than musical instruments to be actually played upon, and, in musical performances, not all the twelve notes were heard in the same composition. This whole series of notes simply enabled the musician to transpose compositions into different "keys"; and, as all European writers know, ancient Chinese music was written in a scale of five notes selected from the whole series of twelve.

It was undoubtedly from musical experience rather than from any a priori principles that the ancient Chinese decided to adopt the pentatonic scale, a simple but pleasing scale of five notes which they called: kung, shang, cho, tze, and yui, corresponding to c, d, e, g, and, a in the western diatonic scale. The intervals between the various notes are: from the first to the second there is an interval of two semitones; from the second to the third,

two; from the third to the fourth, three; and from the fourth to the fifth and last, two. The interval between the fifth note and the octave is three semitones. The psychological and esthetic basis for the universal adoption, East and West, of the pentatonic scale is easily demonstrated.

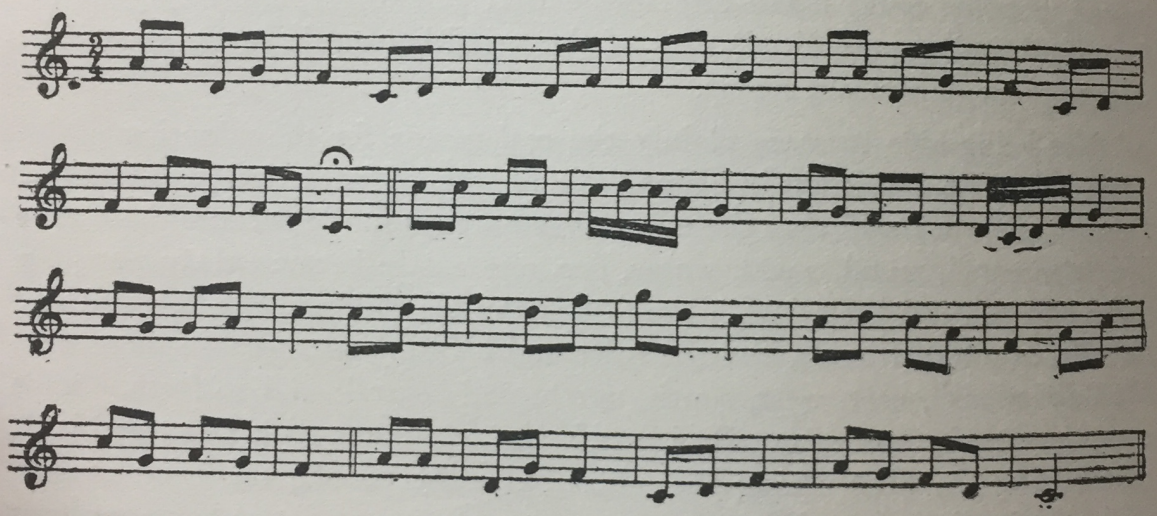
While the original Chinese scale is not in tune with the modern European, with respect to absolute pitch, the tonal relations between the notes in the Chinese are, for practical purposes and in actual playing, essentially the same as those represented by the "black key" notes of the western piano. Without going into the mathematical details of the thing, we can summarily say that such an aggregation of notes has the highest degree of "tonal unity" within a relatively short "vocal span." As we know, the greater the tonal unity among a group of notes the more pleasing combinations they will make. We also know that the purpose of a music scale is nothing more than to make the task of composition less difficult and hazardous by making an arrangement of tones fulfilling this psychologial and esthetic condition. With the pentatonic scale, indeed, nobody could make any music that is entirely unpleasing to the (untrained and unprejudiced) ears of the primitive music lover, even if he tried to make unpleasant music.

Not being able to state plainly the real reason for the adoption of the pentatonic scale, musicians in China gave all sorts of fantastical justifications. For example, they said that there were five notes in music because there were five "elements", metal, wood, water, fire, and earth; because there were five social and political "relations", the father and his children, the husband and his wife, brothers, friends, and the ruler and his subjects; because there were five "directions", east, west, south, north, and central; and so forth. Some even went so far as to actually identify the five notes with five "political" terms, kung the king, shang the official, cho the people, tze governmental affairs, and yui wealth. When kung was out of tune or not in harmony with other notes, misconduct of the king was supposed to be the effect or the cause. And similarly the rest of the story went. Are the Chinese more to be blamed for such metaphysical follies than other nationals for similar ones?

In the construction of the pentatonic scale the principle of the ratio 2:3 held good. Thus, yui, being identical with nan-lui (in the twelve note series) is two-thirds of (i. e., a fifth above) shang which is the same as ta-tsou; tze was produced by two-thirds of kung, etc. Kung and huang-tsung are two names for one and the same note; when it is in the twelve note series, it is huang-tsung, when in the pentatonic scale, kung, which is always the key-note of the scale. Thus, by identifying huang-tsung with kung, we have huang-tsung-kung, or, translated into western phraseology, "the key of huang-tsung or c". However, it is not necessary that kung, the key-note, be always identified with huang-tsung. As a matter of fact, it was identified with any one of the twelve notes in the series, on different occasions. Kung, therefore,

for example, may be identified with ta-lui, the second note of the Lin Lun series, resulting in the transposition of the scale a semitone higher than in the previous case. The key of ta-lui is consequently a semitone higher than the key of huang-tsung. Any pentatonic composition could be transposed into any one of the twelve keys by identifying the key-note of the composition with any one of the twelve notes of Lin Lun. This arrangement was really imperative in ancient China, for in court and religious performances the same composition had to be played in a different key in the different months in order that human music could "harmonize" with the musical constitution of astronomical nature. The key of ta-tsou, for example, must be employed during the first month of the year, the key of cha-tsung, during the second, etc.

The first instrument especially designed for playing pentatonic music is reported to have been a string instrument made by Emperor Shun (2235 -2204 B. C.) known as wu-shien-chin (five-stringed-harp), or pentachord, on which he composed and played the famous song of the "South Wind". The



melody appended here is an excellent, and perhaps the oldest, specimen of Chinese pentatonic music.[38]

The Chinese pentatonic scale has been somewhat misconceived by Helmholtz in his valuable book *Sensations of Tone* (trans. by Ellis, pp. 259-260). Helmholtz holds that the Chinese pentatonic scale was made without Third or Seventh and, when translated into European notation, would approximately be represented by the relations expressed by c — d — f — g — a — c, with three semitones between the second and the fourth tones and between the fifth and the octave. But this is not the case with the original Chinese pentatonic scale. As aforesaid, the pentatonic scale was formed out

38 To understand the Song of the South Wind, the western musician ought to accompany it with the sustained note (drone bass) F or with the sustained notes (drone bass) F and C. The song, ending on the fifth of what is psychologically its key-note, F, ends with "a question," if we take it as a piece of "program music". (M. F. M.)

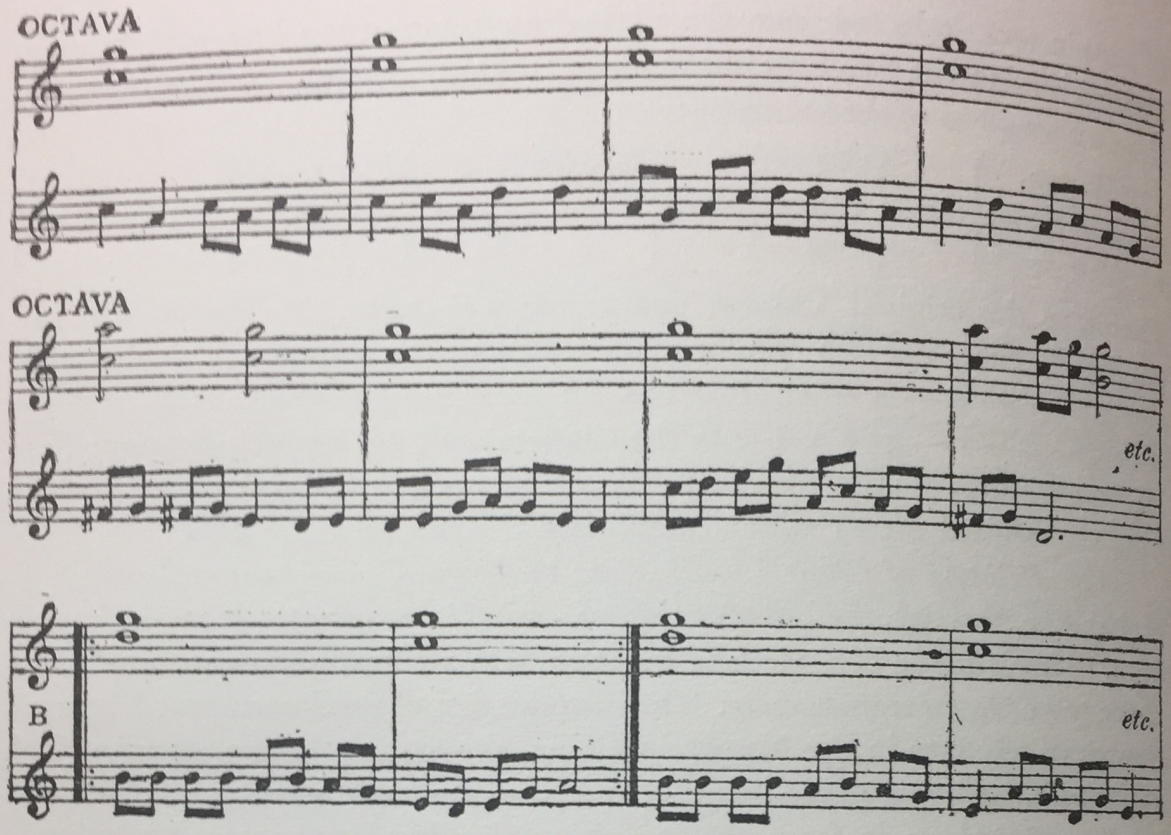
of certain tones selected from the twelve semitones of Lin Lun with different names given to the five tones of the scale. Thus, if huang-tsung was used as king, we would have the following:

$$\left\{\begin{matrix}\text{Huang-}\\ \text{tsung-Kung}\\ \text{c}\end{matrix}\right\}\text{–2S–}\left\{\begin{matrix}\text{ta-tsou}\\ \text{shang}\\ \text{d}\end{matrix}\right\}\text{–2S–}\left\{\begin{matrix}\text{ku-shien}\\ \text{cho}\\ \text{e}\end{matrix}\right\}\text{–3S–}\left\{\begin{matrix}\text{ling-tsung}\\ \text{tze}\\ \text{g}\end{matrix}\right\}\text{–2S–}\left\{\begin{matrix}\text{nan-lui}\\ \text{yui}\\ \text{a}\end{matrix}\right\}$$

Therefore the original Chinese pentatonic was more like the Scotch scale having the character of a "major mode" without the Fourth and the Seventh: c — d — e — g — a — c. However, the fact that Helmholtz attributed the scale c — d — f — g — a — c to the Chinese need not be entirely groundless. He was perhaps misled by the deviation of present Chinese music practice from the original theory which established the pentatonic system.

The *deviation of Chinese music from the original theory* came about, in all probability, through the introduction of a wind instrument, tih, or the keyless flute. This is made of a piece of bamboo with six finger holes and an embouchure to be blown transversely. The introduction of this instrument happened comparatively late in the history of China, the first authentic record of such an instrument being made in the first half of the eighth century A. D. The tones produced on it from the lowest to the first octave are theoretically g, a, c, d, e, g', with the notes foreign to the pentatonic scale omitted here. This instrument, it must be remembered, was tuned by transferring the tones of an older instrument, hsiao, also a keyless wind instrument, played like a flageolet, of higher absolute pitch. The scale on this latter instrument is c, d, e, g, a, c', which is the typical pentatonic scale. (The hsiao can produce all the notes in the heptatonic scale which we shall discuss a little later in this paper.) Now if a person is acquainted with the ancient Chinese theory and the origins of the keyless flute, he has no trouble in locating the key-note of the pentatonic scale on that instrument. But when he does not know that he has to produce the first note of the scale by opening the third hole and the auxiliary holes on the flute, he naturally will attempt to do so by blowing out the lowest note on it, which is g, a fourth lower than c. The scale thus formed will therefore be: g, a, c, d, e, g', seemingly without a Third and without a Seventh, believed by Helmholtz to be the Chinese *theoretical* pentatonic scale.

Unfruitful attempts at systematization were made by subsequent musical theorists at long intervals of time. Among these the effort of a Han Dynasty official, Ching-fang, was the most important, although he made in fact no material contributions in the development of Chinese music. Ching explained to Yuen-ti, an emperor of the Han Dynasty reigning 48-32 B. C., the principle of two-thirds employed by Lin Lun to evolve the ancient system which very few musicians at that time understood. He further proposed to enlarge the original system by increasing the number of notes from twelve to sixty. As we have seen before, the Lin Lun system was supposed to have been constructed by starting from the note huang-tsung and by applying to it the principle of



SAMPLES OF CHINESE ORCHESTRA MUSIC

two-thirds (and four-thirds) until the note chung-lui was derived, completing the cycle of fifths. Ching's idea was that there should be no such cycle; that is, that the series should not stop with the note chung-lui, but should go on from that note to a thirteenth[39] note, chih-shih, produced by making a tube four-thirds of the length of the chung-lui tube, a fourteenth, chu-mieh, two-thirds of chih-shih, and so on, until the sixtieth note was reached. Of course, the span of the whole new series, like that of the original series, did not exceed that of an octave. What he had really done was to introduce several, not more than six or less than four, intervals into the twelve semitone intervals of Lin Lun, dividing the octave span into immeasurably minute intervals, perceptible only with difficulty by the trained ear, utterly impractical in artistic life. In spite of all that, the pentatonic scale was left untouched. His folly is not as great as that of musicians who want to play these little intervals in a piece of music.

All these sixty notes were tones out of which different pentatonic scales might be formed, taking any note as the key-note. Consequently, instead of twelve, there were sixty keys of the pentatonic scale. There were no material improvements made upon the Lin Lun system, except that more possibilities of transposition were created. On the other hand, Ching-fang was indeed successful in making music a hopelessly complicated matter to think about, suitable only for the fancies of the academic mind.

39 Compare footnote 23.

About 430 A. D. another academic musician named Chien Loh-tze proposed to enlarge the sixty-note series of Ching-fang to a 360-note system. The reason for the enlargement was again fanciful. He argued that since there were 360 days in the year, why not make the system consist of 360 notes so that one note would represent each of the days of the year and all musical compositions could be played in a particular key on a particular day? He no doubt was very proud of his remarkable advancement over the ancients who had to be contented with only twelve keys, one key for each month. But his efforts were no more fruitful than those of Ching-fang and naturally he had to suffer the same disappointment of not being approved by the emperor and musicians in his proposals.

II. The Heptatonic Scale: Introduction of foreign elements into the Chinese musical system

The pentatonic scale of the Chinese was modified into a heptatonic, namely, a scale of seven notes with intervals different from those of the European diatonic scale, although the pentatonic idea is still prevalent at present in China. We do not know how this modification came about; perhaps it was more the result of foreign influences (from the Greeks?) than of native ingenuity. In this new heptatonic scale two new notes were added to the original five, not conceived of as entirely new or independent notes, but as modifications of, or deviations from, two of the notes in the pentatonic scale. Thus, the new note pien-tze was considered as a modification of the original note tze (pien, in Chinese, means to change or modify) being a semitone lower than the latter; pien-kung, the other new note, was a modification of the note kung, being similarly a semitone lower than kung. The heptatonic scale is then like this:

Kung	—	shang	—	cho	—	pien-tze	—	tze	—	yui	—	pien-kung	—	kung,	or,
c	—	d	—	e	—	f♯	—	g	—	a	—	b	—	c'.	

The heptatonic (eastern) scale differs from the diatonic (western) scale therefore only in this: that the former has a sharp f, forming an augmented fourth with the key-note, whereas the latter has a natural f, forming a perfect fourth with the key-note. The Chinese, wanting to recognize the value of the introduction of these two new notes, were nevertheless reluctant in employing them in ceremonial performances, for they perceived that the employment of either or both of them imparted a "minor" or melancholic character to the music.

In 1279 A. D., the Mongols, headed by Kubla Khan, conquered China and introduced f natural into Chinese music. Seeing the difficulty of inducing all the Chinese musicians to abandon the sharpened f, the foreign ruler decreed that both the sharp and the natural f were to be retained, although not both

of them were used in the same composition. The Chinese scale at that time was therefore like the following:

c — d — e — f — f♯ — g — a — b — c'.

However, in the Ming Dynasty (1368-1643 A. D.), established by a Chinese conqueror, Chu Hung-wu, who drove the Mongols out of China, the pentatonic scale from which all semitone notes were eliminated was again officially adopted for court and ceremonial performances as a reaction against the influence of foreign music, a reaction caused by the hatred toward the vanquished Mongols, whose rule in China was by no means beneficial. In spite of it the heptatonic scale remains to the present day side by side with the pentatonic, and both have become so familiar to the uninquiring Chinese musicians that practically nobody ever notices that there are two different scales employed in Chinese music compositions.

III. Deviations of Chinese Music Practice from Theory: The Neutralization of the fourth and the seventh of the heptatonic scale. The scale on the keyless flute

The above are the main stages of change in the music system of China. But we should realize that it gives account only of the *theory* of Chinese music, which is different from practice. Nowadays, if we care to study Chinese music practice, we find that the scale produced on the existing Chinese instruments has strangely deviated from the theoretical scales, and this deviation is perhaps the chief cause of confusion of a European student of Chinese music.

This deviation can be best and most easily shown on the tuning of the existing wind instruments, especially the tih, or keyless flute, which has been referred to in an earlier part of this paper, because with the alteration of the tuning of the wind instruments, all the string instruments have to follow the same course of change. It is a well recognized fact that European listeners to Chinese music invariably are impressed by the peculiar character of it, commonly described by such phrases or terms as "queer", "out of tune", or "indifferent", that is, having neither the character of "major" nor "minor" music. That this peculiar character of Chinese cannot be wholly attributed to the difference in theoretical scale construction is obvious if we remember what has been said about the pentatonic and heptatonic scales of China. The original pentatonic, to repeat, is equivalent to the Scotch pentatonic scale, or the scale represented by the black keys of the piano. Although black key music is not commonly heard in western countries, yet no western music lover would ever think Scotch pentatonic melodies, such as "Auld Lang Syne", queer. Even the heptatonic scale with a sharp f cannot sound queer, although such a scale cannot sound "major" to any one accustomed to the diatonic major scale.

We must therefore seek the real cause of the queerness in other factors

than scale construction. The true cause seems to lie in the incorrect intonation of modern Chinese instruments, incorrect in the sense that the intonation has deviated from the theoretical intention of the scale construction.

Let us take up the keyless flute and find out what sort of scale it produces. We choose to take up this particular instrument, because it has long been used as the leading instrument in Chinese ensemble playing, and virtually is the standard pitch pipe[40] of Chinese orchestras, to which all other instruments are tuned. Any deviation in tuning of the flute must necessarily result in a similar deviation in all other instruments. By careful experiment on such a flute, it has been found out that the fourth and the seventh of the scale thereon are both "neutral", that is, the fourth is neither a half-tone above the third as in the European diatonic major scale, nor a full-tone above it as in the theoretical Chinese heptatonic scale, but is just about half way between the third and the fifth, and the seventh is similarly mid-way between the sixth and the octave. It becomes at once clear why Chinese instruments sound impersonal or neutral and out of tune. They would be out of tune to the Chinese, if a musician of several centuries ago who was acquainted with the true pentatonic or heptatonic music were to live and hear them.

The true reason for the incorrect intonation of the existing Chinese instruments cannot be stated with absolute certainty. The most probable reason is this: During the Yuen Dynasty both the natural and sharp f were used, as has been mentioned above, either for ceremonial purposes or for entertainment. But the production of both these chromatic tones, f and f♯, could not be done on the flute except with the assistance of special key devices unknown to the Chinese instrument makers or with boring the second and the fifth holes, counting from the lower end, larger than the rest. It was possible at that time when both f and f♯ were in use to bore the holes on the Chinese flute so as to enable the player to produce these notes, when desired, by a definite system of fingering such as is now used by western fife players. But with the coming of the Ming Dynasty, this whole system transgressing the pentatonic scale was officially abolished.

Musicians then and therefore did not have to bother at all about producing f and f♯, and following the law of supply and demand, the flute makers undertook to make flutes that could not produce either f or f♯ accurately, but *had more uniformity and symmetry in their appearance* by boring all the finger holes of the same size and with a minimum variation of distances from each other. The six holes were there. If a player who had bought the flute cared to

40 Owing to the absence of a universal standard pitch and to mechanical inaccuracies, no two Chinese flutes picked up at random agree in pitch. A Chinese orchestra has to buy all its wind instruments in one shop, or perhaps has the whole set made to order, in order to insure agreement in pitch. "Standard pitch" here means the standard for that particular orchestra.

produce all the tones on the instrument, what he got was naturally a scale with neutral fourth and seventh, agreeing neither with the European diatonic (with the perfect), nor the Chinese heptatonic scale (with the augmented fourth). The same thing was then applied to the playing of stringed instruments, for example, the two-string instrument, hu-ching.

This instrument is played with a bow suggesting the manner of 'cello playing, and the two strings are tuned to a perfect fifth, that is, g and d. The scale is produced this way: g, open string; a, first finger; b, second; c, third; d, open (second) string; e, first finger; f, second; g' third. Now, according to the European diatonic or the Chinese heptatonic scale, there is only a semitone between b and c and in playing, the second finger must stop the string at a place very close to the third. Similarly, according to the Chinese heptatonic scale, f is sharp and the second finger must also be placed very close to the third, and according to the Mongol scale, f natural must be produced by stopping the proper string with the second finger close to the first. *But watch what the Chinese musician does.* He places his second finger, in every case, instead of close to the first or to the third as described above, just about mid-way between the first and the third in order to be able to produce the neutral fourth and seventh *to be in tune with the wind instruments.*

While the writer does not claim that the above explanation offered for the peculiar character of Chinese music is absolutely indisputable, he is nevertheless convinced of its truth. Altho the Chinese are on the whole very patient and painstaking in producing works and objects of art, they assume an entirely different attitude toward the art of music. Imperial patronage of music in the past only caused musical sterility and stagnation which in turn caused a total lack of interest on the part of the democratic people in this art. Nowadays, in certain localities of southern China, the performance of music has come to be looked down upon as a menial and unscholarly vocation; and amateur musicianship is discouraged among persons belonging to good society. Although this aversion is not universal in China, music has been intellectually neglected for a long time. As a result, music has become unmusical in China. And this in turn has increased the contempt in which it is held by the educated classes. It is a vicious circle.

In conclusion it may be said that the pentatonic scale based upon the Lin Lun system of twelve semitones is the only real and original Chinese scale. The heptatonic scales with the natural or sharp f seem to have been formed under the influences coming from people outside of China, and they are, therefore, not originally Chinese. It should also be noted in passing that with the introduction of the heptatonic scale transposing in the original Chinese sense, or in the present European sense, died out. With the heptatonic scale there are only seven keys, using any one of the seven notes as the key-note. The twelve key-notes of Lin Lun are gone. The different keys are no longer transpositions

for they do not have the same intervals between the same degrees of the scale:—

Key of c: c — d — e — f♯ — g — a — b — c'
Key of d: d — e — f♯ — g — a — b — c'— d'
Key of e: e — f♯ — g — a — b — c'— d'— e', etc.

(The original Chinese heptatonic scale is used here as illustration for the sake of convenience.)

The Chinese made a rather brilliant attempt at music at the very start. But China's creative genius was soon crushed under the burden of conventionality and intellectual superstition, and it is high time now for her to start afresh to regenerate in order to save herself from the blame of being a musicless or unmusical nation.

APPENDIX 7. QUARTERTONES IN THE HISTORY OF WESTERN MUSIC

When did it first occur to anyone (expressing his opinions publicly) to suggest the possibility of introducing *quartertones* (*lesser* fractions we do not mean; they were fashionable in the sixties) into our western music? Of course, the question means: introducing them seriously, that is, without doing violence to the basic and essential principles governing all melodious (i. e., better than drumming-roaring) music and consequently also governing western music? If the answer which I tentatively give here is wrong, I wish somebody would correct it.

In Alex. J. Ellis's translation (1895 edition) of Helmholtz's *Sensations of Tone* we find on page 526a, among the "Additions of the Translator," various remarks about the actual existence of quartertone intervals in the scale of *exotic* music. But we do not find Ellis (or Helmholtz) making anywhere the slightest hint that quartertones *could* be introduced into the *western* music system without fear of destroying something at least that is really *essential and scientifically* (*logically*) *correct in the western system.*

On the contrary, it appears that Ellis (as late as 1895) wishes distinctly to reject the idea that any kind of place *could* be found for quartertones in the western music system. Thus, at least, I interpret the following quotation, his brief and clear statement about exotic or "non-harmonic" (as he says) music of easterners. By the way, his definition of "non-harmonic" seems to beg the question whether exotic music is not harmonious; but let us abstain from criticizing him on that score.

Ellis's rather dogmatic refusal, on page 526, as a decent western musician to accept in any way the company of quartertones, is expressed in the following terms: "There (i. e., in exotic music) is an entire absense of tonality in our sense of the term, and of any attempt at harmony." That he, or some other westerner, might get up courage enough to make *an attempt* at harmony and, if successful, *perhaps prove thereby* the presence of some "tonality" in some exotic music, such an idea never occurred to him even as a possibility. The honor of having taken the initiative in this does not belong to him.

Surely, Ellis would not have spoken so tersely if he had known in 1895 of any western musician trying to use quartertones; or if he had wished to forecast the impending introduction of quartertones into our western music system.

My own theoretical and experimental (that is, made in my laboratory) investigations led me to publish more than twenty-five years ago two articles in two scientific journals, in which I demonstrated the theoretical and actual (in the music room) possibility of harmonizing existing exotic quartertone music and making it acceptable to the ears of hearers accustomed to our

usual music.[41] To carry on my experiments I had to construct a harmonium on which one could play the quartertone music discussed in these articles. This harmonium, built in 1902, which uses two manuals, is still in use in the psychological laboratory of the University of Missouri. It was placed on exhibit as a part of the exhibit of the University of Missouri at the Louisiana Purchase Exposition in St. Louis, and for it the administration of the Exposition awarded me a bronze medal which at this moment is lying before me on my desk. Mr. Robert R. Kern, who, then a student, enthusiastically aided me in tuning the harmonium, is now professor of sociology in the George Washington University of Washington, D. C.

In the latter of the two papers just cited, the *harmonization* of a Japanese song can be found which is reprinted in Appendix 2 of the present treatise. In the former of the two papers, also published in 1903, I stated clearly my warning that by "quartertone-music" one must not necessarily (and indeed usually not) mean music which is *melodically* made up of quartertone intervals following each other, like the single beads strung on a string, from one end of the string to the other. Further, this instrument was not tuned in tempered, but in arithmetically exact intonation, so that we could experiment on quartertones strictly speaking as well as on various lesser intervals.

From the report of numerous subjects assisting me in 1902-03 I want to reprint here as very typical a part of that of Professor T. Carl Whitmer (now of Pittsburgh), the second part describing the repetition of the experiment after two weeks: "It is music not familiar to the average ear. The oftener it is played, the more beautiful it becomes." And two weeks later, when the participants had been told that this was quartertone music: "Undoubtedly the music becomes much more beautiful on acquaintance. The new sensations of the quartertones become most agreeable with frequent repetitions." Professor Whitmer later introduced a few such tones in some of his published compositions, and if his use of quartertones is not actually the first in large western compositions, it is certainly one of the earliest attempts at using them in the concert rooms of western countries.

I remember very distinctly that, when I corrected the proofs of those two articles of mine published in 1903, my fingers trembled because I realized the risk to which I exposed myself in *asserting* (theoretically) that quartertone music *could be justified and harmonized* and that it was not "something out of tune;" and *reporting* (experimentally) that such music *had positively been enjoyed* by western hearers to whom I gave a chance to hear it.

I trembled, as reported in the previous paragraph, because I feared in 1903 that many, if not most of my readers after a brief glance would decide that this must be an article written by an insane person. Today quartertones

41 "Experimental Studies in the Psychology of Music," *Amer. Journal of Psychology* XIV (1903), pp. 192-214. And: "Zur Theorie Japanischer Musik," *Zeitschrift für Psychologie und Physiologie der Sinnesorgane* XXXIII (1903), pp. 289-306.

are so much in vogue that there is no such risk. How quartertones have invaded the public concert halls may be learned from a very well written article entitled "Quartertones—and Less" by Grace Overmyer.[42] Only two statements made by her seem queer,—first this one, made with too much naiveté: "There are no records of fractional tone experiments in the United States earlier than 1924." With respect to this date compare the previous paragraph. The second queer statement (and yet its logic may be unassailable) is this: "Busoni is generally credited with having been the first European to experiment with divided tones." Busoni's propositions concerning "divided tones" were published in 1907 in Germany.

Toward the end of his booklet,[43] which is a literary essay and not a scientific treatise, Busoni leads up to his "novel" idea by the following metaphysical-mystical contemplation: "The harmony of *today*, and not for long; for all signs presage a revolution and a next step toward that 'eternal harmony'. Let us once again call to mind, that in this latter the graduation of the octave is *infinite*, and let us strive to draw a little nearer to infinitude." The reader, thus suitably prepared for something revolutionary, is then asked to give his consent to dividing the octave span into eighteen equal parts instead of the customary twelve equal parts. Busoni again: "The tripartite tone (third of a tone) has for some time been demanding admittance, and we have left the call unheeded." Busoni then tells us that he has "experimented" with these eighteenth-octave intervals on the voice and the violin (not on any instrument where every string has a fixed tuning, like the piano) and reports as the outcome of his "experiments" (whose accuracy seems dubious) that the eighteenth-octave intervals have proved to "form a refinement in chromatics based, as at present appears, *on the whole-tone scale*." No scientific definition, however, that is, no definition in arithmetical terms, in ratios, is given of "the whole-tone scale." The whole tone scale mystically appears as an ultimate something, an axiom,—which exactly it is not.

As an afterthought, nevertheless, Busoni declares his unwillingness to give up "our minor third and perfect fifth." Therefore he recedes from his proposed tripartite division of the tone; and, drawing several degrees "nearer to infinitude," he at once "arrives at a system of whole tones divided into sixths of a tone; and *we may be sure* that even sixth-tones will sometime be adopted into musical speech." No condition exists for it, he says, but "a continued training of the ear."

According to Busoni the possibilities of ear training are psychologically unlimited. The biologist must deny this.

Just why we should be psychologically so sure about the advantage of

42 *The American Mercury* XII (October, 1927), p. 207. I asked the editor to correct the erroneous statements made; but he did not comply with the request.

43 In English translation *Sketch of a New Esthetic of Music*, Schirmer, 1911. See pages 30 and 31.

dividing the octave span into 36 equal parts, is not made clear. Theoretical arguments and practical demonstrations are both conspicuously absent from this New Esthetic. Instead of such material support, Busoni gives his flight toward "infinitude" the following spiritual support: "And what a vista of fair hopes and dreamlike fancies is thus opened. Who has not dreamed that he could float on air? and firmly believed his dream to be reality?" I add the question: "Are musicians naturally mystics?"

One thing, however, ought to be made clear; and it is this: Busoni makes no recommendation favoring the use of quartertones, indeed *he does not say one word about quartertones*. This is not to be wondered at. It is plain logic. He cannot have anything to say about quartertones, for in his flight toward "infinity" he asks the next following human generations, for the sake of *gradual* ear training, *to stop over* at dividing the octave into thirty-six equal parts. That procedure damns quartertones, which are twenty-fourths of an octave span, for a twenty-fourth can in no way be composed out of an integral number of thirty-sixths. Why, then, do musicians ever mention Busoni in connection with quartertones?[44] Busoni, who by implication condemns them!

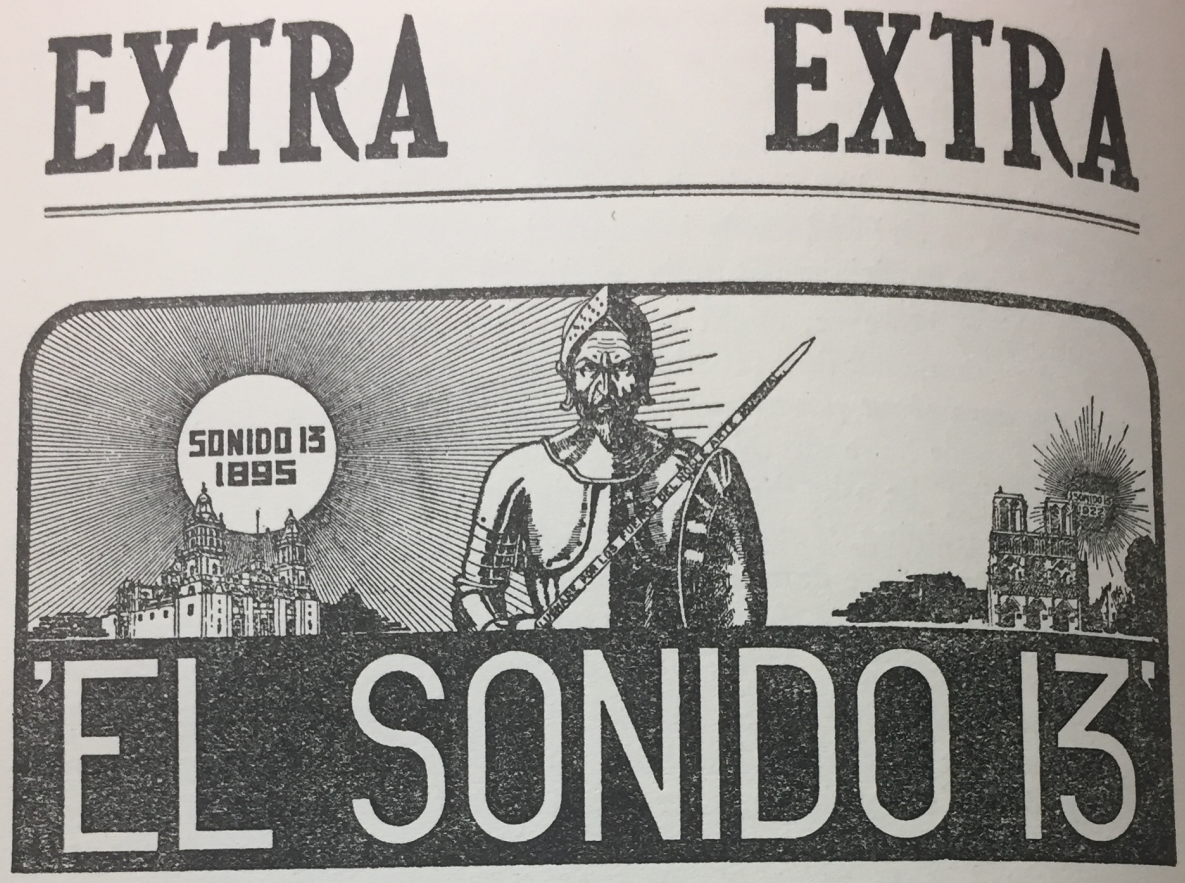
Noticing the haze which had been thrown into the minds of many musicians by Busoni's mysticism, I prepared in 1911 an illustrated lecture on Ethnic Scales and Their Harmonization[45] and delivered it in Ann Arbor at the annual meeting of the Music Teachers National Association. The characteristic conclusion drawn in this lecture and demonstrated on my harmonium is contained in the sentences here quoted (from page 165): "If we desire to enlarge musical practice, this ought not to mean that we wish to break down our present practice. *Only one method* is feasible, then. We may divide the octave into *twenty-four* equal parts. The production of music in ethnic scales would thus require our musical practice to make provision for quartertones. . . . I shall now play you a simple tune in the ethnic scale just shown." What, then, does Miss Overmyer mean, saying; "There are no records of fractional tone experiments in the United States earlier than 1924?" Where is Ann Arbor situated if not in the United States?

In the yearbook of the same society for its St. Louis meeting[46] there is an article of mine bearing a title which I quote for its significant phrasing: "Quartertone Music, its Possibilities and Limitations." In the article is a full page halftone reproduction of the quartertone keyboard of my design, totally different from any such keyboard designed elsewhere in the United States, Europe or any other country.

44 The only psychologically valuable endeavor I have been able to discover in Busoni's "New Esthetic" is his defense (p. 15) of the aria of the old opera, and his other remarks about theater music.

45 M. T. N. A. *Proceedings* VI (1911), pp. 156-168.

46 M. T. N. A. *Proceedings* XIX (1924), pp. 48-52.



CARRILLO'S REVOLUTION, FROM MEXICO'S CATHEDRAL TO NOTRE DAME DE PARIS

Of Julián Carrillo, who, being a Mexican, belongs to the western part of the world, it must be truthfully said that for more than thirty years he has struggled heroically to introduce into western musical practice a greater number of tones than the customary twelve per octave and thereby to "revolutionize" our music, to use his own phrase. But his efforts have been confined to composition and execution, to the exclusion of theoretical understanding; and to him applies in Miss Overmyer's title "Quartertones—and Less" the second part, for he divides the octave into fractions *less* than a quartertone, not stopping at quartertones. He has made no clear public statement contributing either to the *theory* of quartertones or to a *staff improvement* permitting to write quartertones with ease on any staff or to the *design of a keyboard* permitting the easy execution of quartertones. His use of the vague term "el sonido trece" appears unfortunate in science and in art because it has all the aspects of branding an article for industrial and commercial exploitation.

APPENDIX 8. TONALITY AND VOCALITY

English-speaking *organ builders*, and the builders of other musical instruments, have never failed to make *their apprentices* aware of the necessity of controlling each tone in a threefold way. Each tone must be given its suitable *strength*,—or call it intensity, if you speak a more technical language; it must be properly *voiced*; and it must be properly *tuned*.

It is a great pity that those concerned with music should have permitted their ancient and very useful terminology to be dwarfed by the mere accident that our physicists, having for their purposes once agreed on defining a physical tone as a vibratory process in the air, can find in the air only a twofold way of describing what goes on there. The physicists describe a single tone wave only with respect to its *wave length* (or rate, "frequency," which is merely the arithmetical reciprocal of wave length) and with respect to its *wave amplitude*. There is no third property to be found in a single air wave.

But let us remember, there is no third property discoverable by the physicist in light waves either. And yet no one has ever proposed that for that reason the artist and the psychologist in discussing sight should restrict themselves to speaking only of two things, that is, value, which corresponds to the wave amplitude, and a *single* color quality which would vary from the greatest to the smallest visible wave length. The psychologist *has to* speak of *numerous* colors, not of one color, although the physicist has *only one series* of wave lengths undivided by marks of distinction; and the psychologist *has to* speak of a *finite*, very limited indeed, number of chemical processes in the retinal cells, although the wave lengths physically are *infinite* in number. In all this there exists no logical contradiction whatsoever. The relations between the optical and the visual processes are perfectly clear to him who possesses a modern scientific education.

Let us leave sights and remember a few facts about sounds which are quite obvious to the organ builder, the musician and the psychologist.

(1) We speak of the roaring of a lion and the squeaking or piping of a mouse. We can imitate them by means of noise producing instruments. We may say of the noises thus produced that one is shriller than another.[47] The instrument builder tells us that he *voices* such a noise instrument. But he rarely attempts to *tune* it. Why not? We cannot say, i.e., we do not say, i.e., we do not think it worth while to say, that this noise is in unison with the other noise, or that it is its fifth, or that it is an augmented fifth, or that it is a mistuned fifth, etc. If the instrument builder were describing his intentions in phrases like those just used he could not only voice the instrument, *he would also tune it*, tho with probably very little definite success if he keeps it a noise instrument.

47 Compare my article "On the Attributes of the Sensations" in *Psychological Review* XI (1904), pp. 97ff.

(2) When you demonstrate to students very low and very high physical tones, say below 30 and above 8000 vibrations (cycles in modern terminology) you find that quite commonly your hearers refuse to call these sensations "tones." They incline to call them noises. It is wise for the psychologists to say that the instruments used for this demonstration *can not be tuned; but they can be voiced*, for example, by simply turning the piston up or down if it is a Galton (locomotive type) whistle.

(3) We know that it is almost impossible to prevent any single source of sound from producing together quite a multitude of wave length processes. *With respect to voicing*, however, these wave-length processes must be taken psychologically into account in accordance with the amplitude of each; therefore, when a straight brass reed comes from the metal worker to be put into an organ pipe or into the reed cell of a harmonium, it is *first voiced* by bending it and controlling thereby the partial tones, *and then it is tuned* by scraping and controlling thereby its absolute frequency of vibration; and then it is ready to be inserted into the instrument. *Voicing* and *tuning* are two separate jobs requiring each a skill of its own.

(4) When a person, slightly gifted for it by biological heredity as most of us are, tries to learn gradually to acquire an approach to an "absolute memory for pitch," he has to learn to pay much attention to the *voicing* of what he hears. But the tuning does not concern him.[48] For me it is a tremendous achievement when, by listening to the relative shrillness, that is, the voicing, of a piano tone coming to my ear, I can tell that it is neither below the middle c nor above the d next to the right. With *tuning* that has nothing to do, for we do not say that the tone d is mistuned when it is heard instead of c.

From such experiences as the four just mentioned, and from others, psychologists have drawn the conclusion that it is well to have a term for that property of sound with which we occupy ourselves when we are "voicing" an instrument of noise or tone, and another term for that property of sound with which we occupy ourselves when we are "tuning" an instrument of noise or tone. Would not the terms "vocality" and "tonality" have much[49] to recommend them?

In music, both play their roles, tonality and vocality, as we defined them above. *All those effects of music* (call them "emotional" if you wish, altho nothing is gained by the term) which result *from the affinities* found in the variety of pairs of tones and distinguished in this treatise by ratio terms, depend then on the property of tones to be named *tonality*. And *all those effects of music* which can be *illustrated* by a glissando *or* by the usual spreading

48 "Is the Memory of Absolute Pitch Capable of Development by Training?" *Psychological Review* VI (1899), pp. 514-516.

49 "Vorschläge zur akustischen Terminologie," *Zeitschrift für Psychologie* LXVIII (1913), pp. 115-119.

of chord tones, in dance music, on the accented beat and getting together on the unaccented beats *or* by the slow training of a memory for absolute pitch, *or* even by the music-like vowel-qualities of speech,—all these "propinquity" effects, "span" effects, depend then on the property of tones to be named *vocality*. If we want to resort to poetry, we may say that *tonality* is the king of music, *vocality* the queen.

Without tonality the effects of music would approach those of a siren-like howling. And yet *without vocality* music does not exist and is unthinkable. Let us also remember that there are numerous people among us, "unable to carry a tune," for whom tonality exists scarcely or not at all, who are "tone-deaf" (analogy to color-blind!) and for whom vocality alone constitutes the essence of music. This deficiency is incomprehensible without clearly defined terms such as "tonality" and "vocality."

APPENDIX 9. THE EFFECT OF THE RISING AND FALLING INFLECTION

From the book of a friend and colleague[50] I quote a few sentences describing a very important psychological law of speech: "The Rising Slide is the natural vocal sign of Incompleteness. This principle accounts for the old, old rule of the reading book: 'direct questions, or such as may be answered by yes or no, require the rising inflection'; since, if an answer is really looked for, the sense of the question is incomplete, until the answer is given. Negation, when opposed to assertion, ends upward. Most of the unemphatic syllables in the current melody of matter-of-fact speech are rising slides of the second, producing 'the flow' of the voice. . . . . The inherent meaning of the Falling Slide is Completeness, Finality, Conclusion. The repose of the melody at the end of a matter-of-fact declarative sentence is principally, or at least largely, due to the falling inflection or inflections at that point. . . . . In alternatives, comparisons, and antitheses, the rising and the falling third as a rule oppose each other. Example: 'The prodigal robs his heir' (rising);'the miser robs himself' (falling)."

It is not exactly known what the neurological basis of this effect of the rising and falling inflection (or slide, as Scott calls it) is. Perhaps a peculiar property of the nervous tissue results in an increased "preoccupation"[51] when a lower sound (that is, a lower vocality, since this has nothing to do with tonality) is followed by a higher one, and decreased "preoccupation" when a higher sound (vocality) is followed by a lower one. The result would of course be a growing general muscle tonus in the former case and a waning general muscle tonus in the latter case. The individual must then appear increasingly alert, agile, quick in the former case, decreasingly so in the latter case. Whatever the neurological explanation may be, the sensory-motor fact is as undoubted as it is important, for speech and music.

How the independent fact, of social origin, that both speech and music are more likely to end with an assertion than with a question affects the use of the rising and falling inflection appears from a report of Frances Densmore,[52] who made an extensive phonographic collection of songs of North-American Indians: "In 90 per cent of the songs the final tone is the lowest tone occurring in the melody." Analogous observations could be gathered without difficulty from European folk songs and from the works of well known composers. Vocality in this instance appears as the true indispensable consort of tonality. (Compare Appendix 8.)

50 John R. Scott, *The Technique of the Speaking Voice*, p. 307.

51 For a hypothetical neurological definition of the term "preoccupation" see the author's books *Psychology of the Other-One*, p. 93; *Abnormal Psychology*, p. 49.

52 *Chippewa Music*, II, Smithsonian Institution, Bureau of American Ethnology, Bulletin 53 (1913) p. 4.

APPENDIX 10. THE PSYCHOLOGY OF THE LEADING NOTE

It is customary among musicians to use the term "leading note" in a double sense, sometimes in a broader sense, sometimes in a narrower sense.

In a very broad sense a leading note is any particular note which a composer has put just before the final note because he has reasons to expect the succession of *this* penultimate together with the ultimate to give a particularly strong impression, suggestion, hint, of finality. In a narrow sense, one of these notes is singled out by theorists with a moderate degree of justification and is called in their treatises "the" leading note. The one that is thus singled out is usually the note which stands a semitone below the ultimate note.

The esthetic effect of the leading note in a piece of music has more than a single cause. Each of three different classes of effects may make its contribution. The effect of the leading note will be the strongest, of course, when all three separate effects cooperate. It will be greatly weakened, if not even reversed, when one of the three possible effects directly counteracts another one.

(1) It follows from our discussion of vocality in Appendix 8, that the ending of a melody must lack that element of unity which is synonymous with "approaching unison" unless there is the greatest possible *propinquity* between the last two tones. Accordingly, the composer, other demands not running contrariwise, will choose as the last interval the span of S rather than the span of 2S, this latter span rather than 3S, this rather than 4S, and so forth. The larger the span, the less it is likely to suggest that we are approaching the end or that we have reached the end. Think of the movements of a golf ball toward its hole if you want an analogy.

(2) It follows, further, from our discussion of the *falling* inflection and the *rising* inflection in Appendix 9, that (a) the psychological effect of "finality" in the *usual* sense requires that the leading note be higher than the ultimate note. A drama may end either in a way that "is an end" or in a way that "is no end". The former ending is the more usual, the latter the less usual because it is less likely to attract large audiences. Of course, there are occasions (b) when the composer, like the dramatist, dares to suggest to the hearer distinctly the impossibility of a full satisfyingness of life, when he intends that the music end with a question, or with a doubt, or with a tragedy, which means that the drama really does not end at all except by the force of superior circumstances, by "an act of God," so to speak. In such, comparatively rare, cases the desired total psychological effect may be strengthened by reversing this aspect of the leading note effect, and the music will then end with a rising inflection. (c) Very often, toward the end of a composition, the final note is so frequently and quickly altered with higher as well as lower notes that the effect of the rising or falling inflection is by this procedure virtually eliminated altogether. One might almost say in such cases that the ultimate

note in the melody is "surrounded by" both higher and lower penultimate, antepenultimate and other preceding notes. The ultimate then appears centered instead of being reached clearly upwards or downwards. In a "frozen melody", a chord, this avoidance of a clear rise or fall by centering the ultimate is produced with special ease.

(3) We have already learnt in the body of this treatise that the ending-preference between two tones[53] is on the tone represented by the smaller number term of the reduced ratio; and that this effect is exceptionally strong only under a double condition; namely, when this tone corresponds to a pure power of 2 (number symbol "1") and the other tone corresponds to a rather small number (the *smallest* number symbol then possible is 3).

If we think straight and without bias, it is of course obvious that in this third case, which is concerned with "tonality" in the sense of Appendix 8, the effect of *propinquity* in "vocality" which is mentioned first in the above enumeration of three effects is frequently almost, if not quite, *precluded*. Take the affinity 1 — 3 as an example; it is not realizable by a semitone span or even by a span of two or three semitones, for its minimum span is 5S, next to that comes a span of 7S, next to that 17S, and so forth. Among the affinities which come closer to a semitone are 1 — 15 and 1 — 9. If 15 is chosen to be the lower tone (in which case the *rising* inflection "leads" like a question to 1) the propinquity of one semitone establishes a very high unity in vocality between the last two tones of the music. If 9 is chosen to be the higher tone, the *falling* inflection helps the impression of finality, but the propinquity of 2S establishes a slightly lessened unity in vocality between the last two tones of the music. Fortunately the fact that the esthetic effects depend on a variety of causes (propinquity, falling inflection, smallness of ratio term) does no more harm to music than the variety of possible plots does harm to the drama, unless we mean by harm the unfavorable reaction of certain immature minds to whom every plot except the one they are looking for is "poor" if not "impossible".

53 Compare Paul R. Farnsworth, "Atonic Endings in Melodies," *Amer. J. Psychology* XXXVI (1925). The author summarizes on page 400 as follows:

(1) When the ratio symbol 3 is paired with 5, the difference in the ending-preference is small; but 3 is favored, in the proportion 55:45.

(2) When the ratio symbol 5 is paired with 7, the difference in the ending-preference is small; but 5 is favored, in the proportion 57:43.

(3) When the ratio symbol 3 is paired with 7, the difference in the ending preference is quite marked, and 3 is favored, in the proportion 66:34.

(4) When compared with ratio symbols 5 and 7, 3 apparently gives more finality than 5, and 5 more than 7, in the proportion: symbol 3, 41%; symbol 5, 34%; and symbol 7, 25%.

(5) Including the results of Meyer on symbol 2 (or "1"), and reducing his figures to data comparable with the results of these experiments, the proportions would be: symbol 2, 58%; symbol 3, 17%; symbol 5, 14% and symbol 7, 11%.

(6) There seems to be little sex-difference in the ending-preferences.

To these three main facts concerning the psychology of the leading note there is still one fact to be added. A peculiar phenomenon makes its appearance seemingly without reason. It is conspicuous only when the rising inflection is combined with great propinquity in vocality, as in the cases of "the" leading note of the theorists, for example, when b precedes c'. This phenomenon remains too insignificant to say much about when great propinquity goes with the falling inflection, or when the rising inflection goes with considerable remoteness in vocality, say, beyond the octave span, as such large intervals are not used in melody. It has been experimentally proved[54] that when the music *rises* thru the interval a span is preferred the more (a) augmented within reasonable limits the more it surpasses the span of the neutral third (3½S). And if we accept my extrapolation of the data, it has likewise been proved that the more the span of the music falls short of the neutral third (3½S), the more it is preferred (b) diminished, provided again that the music *rises* thru the interval.

If we apply this rule with strict logic, it follows that we should observe a great tendency to take "the" leading note, say, b leading to c', which is below the final note and falls much short of the critical, experimentally found, point of the neutral third, as close as possible to the final note, diminishing the interval as much as possible, but without, of course, altering the chief tone of the piece, the key-note. That the tendency among musical performers who control their intonation (voice, violin, and so forth) is indeed in accordance with our logic, is a commonplace among musicians. Such performers make this semitone interval as diminished as possible, for example, make the b as high as possible when it leads up to c'.

The interesting fact consists for us, here, of course in the possibility of logically *deducing* this so-called "sharpening" of "the" leading note which makes a semitone upward step. We need no longer regard it then as an isolated, a singular, and like all singular cases somewhat mysterious, fact, but as a fact which has entered our scientific understanding.

54 Stumpf und Meyer, "Massbestimmungen über die Reinheit konsonanter Intervalle," *Zeitschrift für Psychologie* XVIII (1898). See pages 350 (kleine Terz) and 371 (grosse Terz, Quinte, Oktave).

APPENDIX 11. FORMULAS MEASURING ESTHETIC UNITY AND VARIETY

On various occasions in the body of this treatise, for example, in discussing Rameau's famous theorem, we have had to introduce into the discussion a provisional kind of "measurement" of the esthetic value of a certain group of tones in order to compare it, that is, in order to give it a rank, a place of merit, relative to another group or other groups of tones. Of course, by measurement is meant here no more than a quantitative expression, a numerical index or coefficient. Since theorists have not yet agreed on any methods of measuring the esthetic values of a "timeless" melody or harmony, i. e., a group without any reference to time so that melody and harmony cease to be different concepts, it seems to be worth while for us here to consider the possibility of reaching an agreement on a matter so important and to make definite recommendations in this respect.

For centuries it has been the custom among estheticians to regard esthetic values as depending on two factors, unity and variety. We ought to endeavor, therefore, to invent suitable formulas for measuring unity and variety. Since musical tones have two distinct psychological properties (or attributes, as psychologists formerly said), vocality and tonality, it will be our task to recommend for adoption four formulas, of which two measure the unity and two measure the variety.

Vocal Variety. It would be difficult to give any better meaning to vocal variety than some kind of proportion to the greater or lesser number of *different vocal spans* available to the musician who makes music on the musical instrument he happens to have in his hands, and who uses all the tones thus available. There may be several good methods of measurement. Any one of such methods can be pronounced bad only after one of its results has been found to be contrary to recognized musical experience. As long as no such contradiction has been found, provided the method otherwise appears reasonable, logical, it must be pronounced good in a pragmatic sense, that is, convenient. "Good" means then that it is applicable to measuring a value whose *experimental* determination is inconvenient.

In all probability a quite satisfactory measurement of esthetic vocal variety consists in *just counting the number of* (*conspicuously*) *different tones available*, for in general a tone added on either end of a series merely duplicates all of the already existing spans and *adds only one that is new*, i. e., one that is larger than any of those previously existing. Whether more delicate methods of measurement are worth more than any "hairsplitting" method is worth anywhere in life, is doubtful. But it must be admitted to be an open question.

Tonal Variety. One can hardly give any other meaning to tonal variety than the *greater or lesser number of different affinities*. For example, the group of three tones of the actual ratio 8:9:24 (say, C:D:g) would have among them

only two different affinities, since the affinity symbol of 8:24 (i. e., 1 — 3) is identical with the affinity symbol of 9:24 = 3:8 (i. e., 3 — 1), and there is only one other affinity symbol discoverable in the group, namely that of the ratio 8:9 (i. e. 1 — 9). The tonal variety of this group, 8:9:24, is then measured as equal to two units of measurement and can then be pronounced as inferior to the tonal variety of the group, say, 9:10:24, which has three units of tonal variety. Let us prove that 9:10:24 has three units when measured by the proposed method. The affinity symbol of 9:10 is 9 — 5. The affinity symbol of 10:24 is distinct, for it is 5 — 3. And the affinity symbol of 9:24 is again distinct from 9 — 5 as well as from 5 — 3, for it is (9 — 3, or rather) 3 — 1. The tonal variety of the group 9:10:24 is therefore correctly measured, for comparison with 8:9:24, as being equal to three units. This seems to be a very feasible method of measuring tonal variety.

Vocal Unity. The problem of measuring esthetic unity presents much greater difficulties than that of measuring by formulas esthetic variety. Let us agree at once that it is scarcely possible to conceive of unity as being variable toward infinitude or as being ever a negative value. When a mathematical formula has to be devised in such a way that it varies between a definite minimal, but positive, value and a definite maximal value, it is generally most convenient to take the maximal value as 1 and the minimal value as zero. Let us do this here. To measure vocal unity, we should be guided by the very striking fact that composers in general avoid a scale in which the steps from tone to tone vary greatly. The middle voices are usually distributed between the extreme voices so that a fairly even "spacing" results. Irregularity among the vocal spans seem to reduce the vocal unity. Therefore the following procedure seems to be recommendable:

Arrange all available tones in a vocal series and regard the *total span* as composed by the series of *partial spans* from tone to tone, just as you cover the length of a ladder by stepping from rung to rung no matter whether the rungs are evenly or irregularly spaced. Now form all the quotients by severally dividing the measure of the total span (in units of S, for it would be "hair-splitting" to use here cents) by each of the measures of the partial spans. Then add these quotients together.

Now use the square of the number of the partial spans as the dividend of a fraction which is to be formed; and use the sum found by adding the quotients above defined, as the divisor of this fraction. Remember that the square of the spans which we want is $(n - 1)^2$ if n is the number of tones. Use the result of the division (or the value of the fraction, if you prefer to say so) as the measure of the vocal unity. This measure will never be larger than 1.

Let us make an application to four problems. Take a piece of music composed of the tones G — A — c — d — e — g — a. The total span is 14S.

The successive partial spans form the following six quotents: 14/2, 14/3, 14/2, 14/2, 14/3, 14/2. The sum of these quotients is $\frac{42 + 28 + 42 + 42 + 28 + 42}{6}$ or 224/6 or 37.3. This is the divisor. The number of tones is 7. The dividend looked for is therefore 36. And the measure of the vocal unity is in this case 36 divided by 37.3 or 0.97.

As a second sample problem let us take a piece of music composed of C — E — Gis — c. The sum of the successive quotients is $\frac{12 + 12 + 12}{4}$ or 9. The square of (n — 1) is in this case 9. This divided by 9 is 1.

As a third problem take C — D — B — c. The successive quotients are 12/2, 12/9, 12/1; and their sum is $\frac{108 + 24 + 216}{18}$ or 19.3. This is the divisor. The dividend is 9. And the measure of the vocal unity is in this case 9 divided by 19.3 or 0.47.

As a fourth problem take C — D — G — A. The successive quotients are 9/2, 9/5, 9/2. Their sum 108/10 or 10.8. This is the divisor. The dividend is 9. And the measure of the vocal unity is 0.83.

The Product. What we are particularly interested in is of course in each case the *product* of our measure of vocal unity and vocal variety. We find that this product is for G — A — c — d — e — g — a equal to 7 x 9.7 or 6.8. For C — E — Gis — c it is 4 x 1 or 4. For C — D — B— c it is 4 x 0.47 or 1.9. And for C — D — G — A it is 4 x 0.83 or 3.3. These results, compared with musical experience, do not appear contradictory, look quite reasonable.

Tonal Unity. Let us agree, as in the case of vocal unity, that we want a measure which varies from a maximal value of 1 down toward zero. Such a measure can conveniently be obtained in the following way. We remember, of course, that we are now concerned merely with the number symbols of the tones, as defined in the body of this treatise, forming a scale consisting exclusively ot odd numbers and usually without repetition.

Take as the dividend the double number ot the pairs which can be formed of the number symbols of the scale. If these symbols are counted to be n, the number of pairs which can be formed is given by the familiar mathematical formula $\frac{n(n-1)}{2}$; and the double number of the pairs is simply n(n — 1).

Form all possible pairs of the tone symbols in the scale. If any pair, separately, permits reduction of the terms, reduce. Then add all the terms of all the reduced pairs and use the resulting sum as the divisor.

Now divide the dividend by the divisor and regard the result as the measure of the tonal unity of the scale. This measure can never be more than

1. It will be 1 if the scale consists of nothing but octave tones. For example, four octave tones would have $n(n-1)$, the dividend, equal to $4 \times 3 = 12$. The divisor would be 12, too, because the separate sum of the terms of each of the 6 pairs would be 2; and 6×2 is 12.

The Product. In order to see whether the formula works out reasonably, let us tabulate a few sample scales, indicate the measure of the tonal unity, and indicate also the *product* of the measure of the tonal unity and of the measure of the tonal variety.

Scale	Measure of Tonal Unity	Product of Tonal Unity and Variety
1 — 3 — 1	6/10 = 0.60	0.60 x 2 = 1.20
1 — 3	2/4 = 0.50	0.50 x 1 = 0.50
1 — 5	2/6 = 0.333	0.33 x 1 = 0.33
5 — 3	2/8 = 0.25	0.25 x 1 = 0.25
1 — 5 — 3	6/18 = 0.333	0.333 x 3 = 1.00
5 — 3 — 15	6/18 = 0.333	0.333 x 3 = 1.00
1 — 9 — 3	6/18 = 0.333	0.333 x 2 = 0.67
1 — 9 — 5 — 3	12/46 = 0.26	0.26 x 5 = 1.30
1 — 9 — 5 — 21 — 3	20/112 = 0.18	0.18 x 7 = 1.25
1 — 9 — 5 — 3 — 27	20/120 = 0.17	0.17 x 7 = 1.18
1 — 9 — 5 — 3 — 27 — 1	30/169 = 0.18	0.18 x 8 = 1.44
27 — 1 — 9 — 5 — 3 — 27	30/196 = 0.153	0.153 x 8 = 1.22

The data of this table seem to agree well enough with musical experience. Just try it by playing these scales on your piano. All our reasoning proceeds under the assumption, of course, that all the tones of whatever scale is mentioned actually enter into a *perceptual unit* whose esthetic unity and variety we measure with respect to tonality. That all the tones will enter on any

occasion into a perceptual unit, however, is the less likely the larger the scale, due to various reasons among which may be mentioned not only temporal scattering in an *extended melody* but even such a primary factor as *deficient analyzing ability of the organ of sense* with simultaneous tones.

A piece of music as a whole cannot, of course, be measured simply by such formulas. It is composed of smaller units, of "sentences", just as a spoken discourse is not a unit of thought measurable by one simple formula of the logician, but is composed of sentences, or rather paragraphs, separately to be considered. A whole musical "discourse" must likewise be broken up into pieces, phrases, themes, before simple formulas are applied to it, or rather to these separate pieces.

One warning must still be pronounced on this occasion. It is important to remember that our organ of audition functions so that when tones are sounded *simultaneously*, they simply are not always all audible.[55] The formulas above recommended of course refer only to audible tones, not to all the tones which someone supposes to be audible because, say, he strikes so many piano keys together.

55 This disappearance of tones is nowadays referred to by the term "masking". Compare Max F. Meyer, "The Hydraulic Principles Governing the Function of the Cochlea", *The Journal of General Psychology* I (1928), pp. 239-265. See especially pages 254 and 255. The musician is apt completely to overlook the facts there mentioned and to take it, often quite wrongly, for granted that what is written in the score and executed by the fingers of the pianist can be regarded as strictly corresponding to the sensory processes in the organ of hearing. The saving factors, so far as musical practice is concerned, are such as these: What one ear does not hear, the other does; and the physical-acoustic fluctuations (such as the vibrato) are so ordinary that what is inaudible during a short fraction of a second (much shorter than even the length of a short note) is audible during another fraction. Thus, if not all, at least most of what the composer wishes to become audible, does become audible during the time which results from adding all these very short moments together. Nevertheless great caution is needed in assuming that everything is audible which is "played out" in a mechanical sense.

APPENDIX 12. RHYTHM—ESPECIALLY WHAT IT IS NOT

No doubt the statement made in the next sentence will run counter to the way of thinking, the prejudice, of nine-tenths of my readers. Rhythm is *no essential element* of music, neither in the restricted and psychologically alone proper sense of the term "rhythm" as grouped action (that is, a multitude of actions held together by accentuation), nor in its more popular sense, referring merely to things that recur again and again after equally long intervals. Few people ever make clear to themselves that "rhythm" has a double meaning. In the sense widely adopted by "the man in the street," of a strictly regular repetition, we could, and perhaps should, replace the word "rhythm" by the scientifically used term "periodicity"; and yet in that (no longer scientific) sense the word "rhythm" is unfortunately used by all of us in our everyday life, e. g., when we speak of the rhythm of the moon making a month of moon phases, the rhythm of the sun making a day, the rhythm of breathing and of the pulse, the rhythm of the soldiers marching in time in order to keep from stepping on the front man's heels. All this can be described perfectly and far less ambiguously than by speaking of rhythm, by speaking of the *period* which recurs so and so often during a larger time unit, and by mentioning within that period the fractioning point, the *phase*. This is done by science.

That music can be used in order to help a soldier to avoid stepping on another soldier's heels, or to help a dancing couple to avoid stepping on each other's toes, is undeniable. But the pure noise of a snare drum would serve the same purpose. On the other hand any soloist avoids strict regularity of timing; for example, the North American Indian, singing, adapts the length of the notes to the words used.[56] And why should he not? The Indian does not sing to help marchers or dancers to get along better; he sings in order to establish a relation with the Divine Being.[57] And his "rhythm" does not consist in regularity of repetition, but in the *grouping* of his tones. It is like the "rhythm" of *prose*, not like the "rhythm" of metric poetry.

In the narrower use of the term, necessary, if at all, only for scientific purposes, psychologically alone justifiable, "rhythm" means the *grouping* of a number of acts, or of stimuli calling forth acts, by emphasizing one element

56 Frances Densmore, *Chippewa Music* I, Washington, 1910, p. 5: "There are measures of irregular lengths. . . . In other songs there is no recurring rhythmic unit and in many songs of this class the entire melody constitutes a rhythmic unit." And further *Chippewa Music* II, Washington, 1913, p. 15: "In many instances the song is completed and the singer at once begins the repetition, disregarding uniformity of measure-lengths. . . . The note values are changed when words are introduced."

57 Frances Densmore, *Chippewa Music* II, p. 15: "Investigation of the origin and use of Chippewa songs leads to the conclusion that most of them are connected with the idea of reliance on supernatural help." P. 42: "The love songs of the Chippewa are plaintive in character, usually expressing sadness and disappointment. . . . indicative of an unhappy state of mind, 40 per cent of them are major in tonality."

in the group thru making it (on every occasion take your choice!) a trifle *longer than*, or somewhat *more intense than*, or qualitatively *different from*, the other elements which without this emphasis might cease to be a group. The group is then usually and rather naturally, but not necessarily and essentially, i. e., *not always* in the life of a human being, followed by other groups more or less similar. One quite important difference between prose and poetry consists indeed in this: That in prose successive groups are usually only very slightly similar, whereas in poetry the successive rhythms are often virtually identical as in the case of certain metric feet. In music we find what we find in prose and in poetry. The widespread notion that music *must* be composed of groups as alike as the *measures look* on the staff, is merely the result of seeing the written scores of our modern "many-voiced" music. Of course, if musicians are to execute *together* the intentions of the composer of *an orchestra score*, they have to execute the time values of the measures with the greatest exactness as shown by the measure. But it does not follow from this necessity, that *music*, taking the word in the broadest sense, contains a *regularly repeated* rhythm as one of its essential elements. It is probable even that every kind of distinct grouping by accentuation could be absent and still leave a specious group of tones (being a group only because there are no further tones, i.e., because they are all and no more) nevertheless *deserving* to be classed as music.

During the nineteenth century there was among psychologists much *metaphysical* speculation about "rhythm", which was supposed to be one of the properties of the human soul and to help to establish man's superiority over the "animals". This metaphysical speculation among psychologists about rhythm led to an entirely unjustifiable interpretation of the fact that we find but rarely experts in the use of rhythms in which a larger number of elements, *five or seven*, form the group, but that people who can make some practical use of rhythms of the two and three stroke kinds are quite plentiful. Psychologists used to think that *certain* rhythms were "inborn" and *others* did not really deserve to have their existence asserted. This notion is dying out, like so many others about inborn qualities of the human soul, even in the most conservative textbooks for undergraduates.

The practical, the only problem for the modern musician, consists in an answer to the question: How does one acquire skill in easily executing various groups by greatest emphasis on a single one of the elements and lesser degrees of emphasis on other elements composing the group? What is the best method of training for such skill?

We can profit, for answering this question, from observation of the "grouped motions" *acquired by infants*. A universally acquired group motion makes up the act of walking. Everybody must learn to walk. A human being has two feet. He cannot walk using different gaits as a four-legged animal may.

He has to step alternately on one and the other foot; there is no further choice, that is, there is no choice at all. Since virtually every human being is neurologically one-sided (usually right-sided), one of the two feet takes up mainly the heavy. unskilled task of steadily keeping the center of weight of the body *above* a certain distance from the ground; the other foot (generally the right foot) assumes mainly the lighter, but highly skilful, delicate, task of keeping the body balanced sidewise.[58] Accordingly from now on daily, all through life, the individual exercises a group motion of two elements, a heavy and a light one, that is, a "two-stroke rhythm." Everybody has it. This ability to group may be transferred to any other muscle system of the body. (The transference of rhythm is probably the only true "transference of training" in the whole process of education.) It may be transferred to speaking. Thus we understand why in poetry the most common metric feet are the trochee and the iambus, or would be if there were enough two-syllable words. The poet who writes other rhythms writes for a smaller audience at his risk.

The three-stroke rhythm is more rarely acquired by the individuals composing the human race, because opportunities for its acquisition are rarer. Nevertheless such opportunities cannot be said to be extraordinarily rare. Among hundreds of opportunities possible there is mentioned in the book just cited, on page 344, the case of a gardener who has planted two rows of plants, walks along between the two rows and alternately stepping right and left firms the loose soil. This can not be done economically, easily, except by combining every three motions into a group in which one of the three is more emphasized than the other two.

In general, any odd-numbered group of motions is, artificially for training purposes, easily developed out of an even-numbered group of motions by simply changing, from group to group, the direction either in the sense of changing from one side to the other (as the above mentioned gardener does) or from the vertical to the horizontal, back to the vertical, and so forth. On page 347 of the same book is pictured a frame used in my laboratory for *learning* unusual rhythms, such as the five-stroke, the seven-stroke, the eleven-stroke rhythm, by merely striking *systematically* and with an imagined purpose various points of the frame with a mallet. After sufficient practice the particular rhythm habit thus acquired can be neurologically transferred at will to any muscle system other than the one used for practice, e. g., to the speech organs for the enjoyment of unusual metric feet in poetry (altho there seems to be no "five-stroke" poetry as yet), to the fingers for the enjoyment of playing with ease, without having to count, music with five or seven beats to the measure (there are already some such compositions), to the feet for the enjoyment of less usual dancing steps, etc., etc.

58 Compare the more elaborate discussion of the acquisition of special "rhythms" in my *Psychology of the Other-One*, pp. 331 ff.

Further directions for taking such rhythm exercises without counting, and yet taking them even more efficiently than with counting, can be found in the laboratory manual which I put in the hands of my students.[59]

In my previously cited textbook, p. 349, there is an example of an Arabic (semi-religious) processional dance, the music of which is written in five beats to the measure. It illustrates the fact already mentioned that an odd numbered rhythm habit develops unavoidably out of an even numbered one when the performer every time when he repeats has to change the direction. The Arabic dancers have to salute alternately the admiring populace on the right and on the left side of the street. If it were not for that, music of four (or two) beats to the measure would serve them well for the mere forward march or dance. The change of sides introduces a fifth beat to the measure, which marks the heavy step supporting the body when bowing to the spectators, alternately on either side. A Hindu song used by children in India for religious purposes, which contains the seven-stroke rhythm, is quoted by Abraham and Hornbostel.[60]

We have already emphasized that rhythm, being grouping, signifies "irregularity" of the elements entering into groups, and even from group to group, and by no means regularity of intensity, quality or duration (altho the irregularity is never greater than sufficient to do the "grouping"). The musical "measure", which looks like a stern determiner of temporal regularity in the smallest subdivision, is intended to serve this purpose actually only in ensemble playing, especially in orchestra music. Where this requirement of *regularity to avoid confusion* does not exist, music would probably be better off if it were (more honestly) written in no measures at all, but with plenty of phrasing and expression signs.

A very instructive discussion, illustrated with many musical examples of the grouping of tones without forcing thereby upon the tones the strict time requirements which the visual aspect of the measure so strongly suggests, is found in a paper read a few years ago before the Music Teachers National Association. That is, it is instructive if we can emancipate ourselves from the terminology of the author.[61] He speaks (p. 65) of a "contradiction between the apparent rhythm and the real rhythm." What he means is not that the composer uses "tricks" in order to *hide* the "real rhythm," but rather that the composer, writing for the piano, while customarily squeezing his written notes into the apparently rigorous scheme of the musical measure, intends to write simply *one group* of tones to be executed with the left hand and

59 Max F. Meyer, *Elementary Laboratory Psychology, A Brief Manual of Psychology Demonstrations* (1927), pp. 39-45.

60 "Phonographierte Indische Melodien," *Internat. Musik-Gesellschaft*, V, No. 3 (1904), p. 361.

61 Walter Goldstein, "The Rhythmic Tricks of Chopin and Schumann," *Proceedings M. T. N. A.* XIX (1925), pp. 63-73.

another group (not "rhythms" in the sense of "periods") of tones to be executed with the right hand. But how long the notes of each group are to be played, is *not to be figured out* by regarding the written "measure" as a graph exactly describing the duration of each note, but is to be decided by analyzing, thru listening, the melody of the right hand and the melody of the left hand and holding then each note so long as the primary and secondary accents of either group, melody, call for. That is, the *composer* does not wish to write "rhythm" in the sense of measured time, of period, but wishes to write "rhythm" in the sense of "grouped tones."

That the author fully understands the fact that Chopin and Schumann do not intend to write "rhythm" in the sense of period, of a required *temporal regularity*, appears on page 69 when he says: "This *irregularity* is one of the very charming things throughout Chopin's works." But he confuses the reader by bringing in the ambiguous term "rhythm" and saying on page 73: ". . . for those whose rhythmic sensibilities are not so keen as they should be, the deficiency often leading them to take the written page too literally." Here we have to retort as psychologists that "timing sensibility (without a metronome)" and "grouping sensibility (without need of counting)" are two different abilities; and that it is *best* to refer to neither of them by the confusing, the popular, term "rhythm"; *tolerable* to refer to the latter ability alone as "rhythm"; *intolerable* to refer to both (or to the former, timing ability, alone) as "rhythm". Those who "take the written music page too literally" are obviously good timers, but deficient groupers. This way of stating is clearer than ambiguously accusing them of a "lack of keen rhythmic sensibility."

To what an extent the "interpretation" of a composer's music by various artists depends on varieties of *grouping* is made clear in a study of musical style by means of the reproducing piano.[62] The following quotation shows what remains after eliminating differences of intensity and differences in pedaling and differences of tempo:

"There remains only one factor that can be responsible, namely, the temporal disposition of the individual notes within the measures, that is, their *grouping*, or if you like, their *phrasing*. Both pianists depart markedly from the exact time-values of the original score. *The two pianists*, however, do not depart in the same manner, but *differ from one another*. The grace note is played by Friedheim practically as it is written, but Paderewski holds it as long as the next five notes combined, and furthermore adds a grace note in the bass. This difference is one of several that play an important part in producing the marked difference in the total effect. It may be noted in passing that no musician mentioned it when asked to describe the ways in which the renditions differed. We can well imagine, however, that a pedantic piano

62 G. M. Whipple, "A New Method of Analyzing Musical Style by Means of the Reproducing Piano," *Journal of Applied Psychology* XII, No. 2 (1928), pp. 210f.

teacher would reprimand her pupil for being so 'terribly careless' of grace note values."

We have seen to what extent a mutual lack of comprehension may result from our using in music the ambiguous term "rhythm" or "rhythmical sensibility." Let us substitute two terms, "ability to divide time and keep time" and "ability to group," it being irrelevant for this distinction whether grouping relies on duration or intensity or qualitative differentiation for the needed accents.

APPENDIX 13. EQUAL TEMPERAMENT IS NOT A POSITIVE PSYCHOLOGICAL FACTOR

Equal temperament is sufferable because of the enormous practical advantages resulting from its use. But let us not deify it and attribute to it positive psychological powers as long as we have not in vain exhausted all other explanatory factors having the power to produce this or that ordinary or experimental experience. Neither the caveman's *habituation* to the reverberations within his abode nor the modern man's *habituation* to the tempered piano seems to be a needed explanatory factor in psychology.

Thruout this treatise we have been guided by the conviction that the human recognition of similarities in "tonality" (as contrasted with "vocality") between sounds, neurologically depends on chemical affinities which become apparent thru certain simplicities of the vibration rates in "ratios" of stimulation. These chemical affinities we have regarded as inherent in the hereditary biological properties of the nervous system and in no way as the result of "individuals of the human race getting accustomed" to something. At the same time we have accepted the view that the human organism tends to react to a pair of tones *as if* they corresponded to a certain ratio, even in the case of a deviation from that ratio, as long as this deviation is small, with diminishing probability of an equal reaction as the deviation increases.

A view quite contrary to this, however, has for a long time had numerous adherents among musicians and even among a few psychologists,—the view that "we like" any "ratios" simply because we have become accustomed to them. A variety, or subdivision, of this latter view is the following, against which I wish to pronounce a warning in the present appendix: Since we hear "tempered" intervals far more frequently, at least if we are exposed to pianos or other instruments with fixed tuning, than we hear intervals of the "just" (simple-termed) ratios, the actual biochemical affinities in our nervous system belong to these tempered and *not to the just* intervals. Or, in every-day phraseology, "we like," so it is alleged, "the tempered intervals better than the just ones."[63]

Every once in a while someone comes and asserts that he has proved the latter allegation "by experiment." I wish to show therefore by an example how readily that kind of experiment may miscarry in the hands of one who is not thoroly trained in the psychology of tone and music.

At a certain scientific meeting a report was made of the following result of an experiment on the "acceptability" of more or less "mistuned" musical intervals. The experimenter had restricted himself for reasons of his own to working on his subjects with the major third, the fourth, the fifth, and the

63 From this it follows, of course, logically, that the whole theory of "intervals of simple-termed ratios being psychologically *just* intervals" is really a fraud, and that *hereditary* biochemical affinity in "tonality" of sound is non-existent.

major sixth. The result was presented in a table somewhat like the following except that we represent in our table only the relative order of magnitude, not the exact magnitude of each of the deviations.

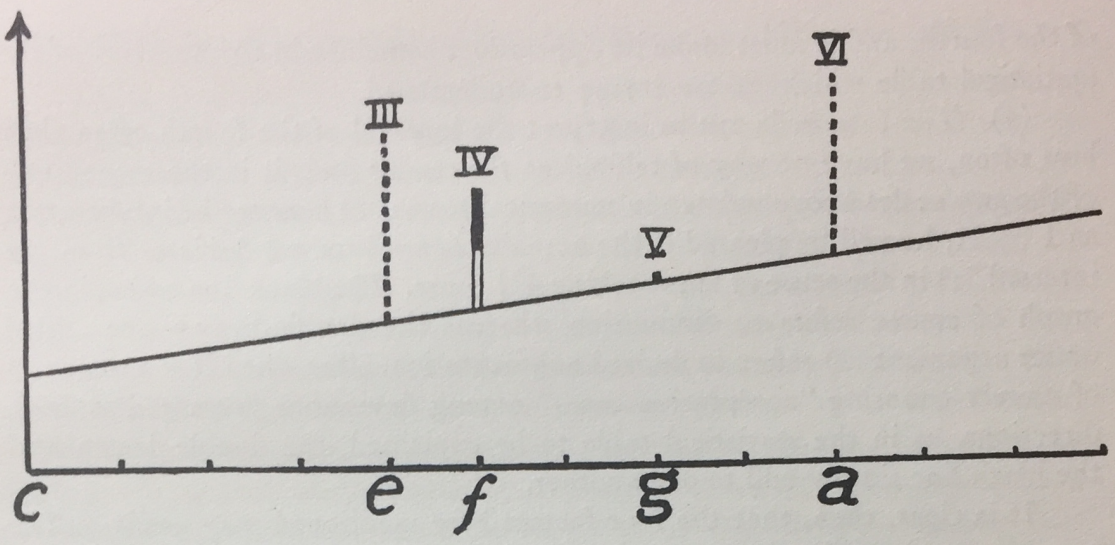
Interval mistuned:	Acceptable often enough with, say,
Fifth, 2 : 3	about 3 cents deviation
Fourth, 3 : 4	about 4 cents deviation
Major Third, 4 : 5	about 5 cents deviation
Major Sixth, 3 : 5	about 6 cents deviation

That a biological organism accepts, within limits, deviations from the norm is of course in no way remarkable or even interesting. What is interesting in the table above is merely the peculiar order of the intervals, the fifth coming first as being apparently "most sensitive" to any encroachment upon its rights, being followed by a smaller and a yet smaller interval, only to have the largest interval make up the end, being "least sensitive." There seems to be absolutely nothing rational in *this* order. And yet the rare person who is familiar with the psychological principles which have been previously stated in this book, and among which habituation to tempered tuning is not found, has no difficulty in showing the reasons for that order of the four intervals by applying those principles, without having to bring in habituation to something which is totally foreign to the hereditary biochemistry of the organism, such as tempered tuning.

If the experimenter had only included the octave among the intervals used, avoiding then of course the *simultaneous* presentation (used by him unnecessarily; compare footnote 16) of the two tones because the mistuned octave would conspicuously "beat", he would have discovered that what is true for the sixth is also true for the octave;[64] and it would never have occurred to him to throw all other explanations into the discard in favor of this, least probable, one, habituation to tempered tuning. An octave on the tempered piano is never heard "tampered with"; indeed, the justness of the octave is the very basis of "temperament" in its definition and in the practice of tuning.

After having pointed out that experimental facts which have been known for thirty years exclude the explanation by reference to the existence of equally tempered instruments, let us apply, in order to understand the order of the four intervals in the table, by means of an approximate and graphical method the following four psychological factors: (1) Weber's Law, (2) The Degree of Consonance, that is, of immediate recognition of similarity or affinity in "tonality" of sound, (3) The Effect of the Leading Note, (4) the obvious Ambiguity of the Interval of the fourth, capable of being interpreted as 9 — 3 (i. e., 3 ↗ 1 = 498 cents) in such a scale as, say, 9 — 5 — 45 — 3 — 27 — 15 — 135, but as 9 — 189 (i. e., 1 ↗ 21 = 471 cents) in the scale 9 — 81 — 45 — 189 — 27 — 243 — 135.

64 Compare Stumpf and Meyer, "Massbestimmungen über die Reinheit konsonanter Intervalle," *Zeitschrift für Psychologie* XVIII (1898), p. 371.



SOMETHING ELSE THAN HABITUATION TO TEMPERED INTERVALS

(1) The slanting line in the graph represents roughly the fact that the alteration of an interval by a given number of cents must be expected, in accordance with Weber's law, to be the more objected to (to be the less acceptable) the smaller the vocal span.

(2) The two vertical and dotted lines which have been placed on the slanting line graphically represent the (relative) rising of the acceptability in consequence of the fact that the third and the sixth are of lesser consonance (lesser affinity) than are the fourth and the fifth. We therefore in the graph add nothing to the ordinate values for the latter intervals, but add an equal amount, the height of the dotted lines, to the former intervals.

(3) The double-drawn vertical line represents an indirect effect of the "power of 2" which is the ratio term for the higher tone. The interval of the fourth is the only interval here present which tends to force the hearer to conceive it as composed of a leading note and a final note which is higher. It is the only interval which tends to make the listener *think of it as a rising interval*, 3 ↗ 1. It is true that the third and the fifth also contain a power of 2 as a ratio term; but the leading-note effect resulting therefrom in these cases makes them falling intervals. As soon as, and whenever now, the fourth happens to be thought of by the listener as a rising interval, 3 "leading up to" 1, the fundamentally human tendency to prefer the interval *widened* becomes considerable.[65] Therefore the double-drawn addition in the graph has not been given to any of the other three intervals, but only to the interval

65 Compare footnote 54 and what is said in the text in connection with that footnote. See also Meyer, "Experimental Studies in the Psychology of Music," *Amer. Journal of Psychology* XIV (1903). For example, page 206: "The preference. . . . is extremely slight in comparison with the same rising interval. . . . This law holds good only for rising intervals." (This includes, however, intervals "thought of" as rising.)

of the fourth; and it must make its appearance somehow in the numbers of the statistical table which we are trying to understand.

(4) The listener is apt to interpret the interval of the fourth often (but how often, we have no way of telling) as the reader finds it in the second one of the two scales above written in numerical terms. Whenever he interprets it as 1 ↗ 21, he will be pleased if the actually heard interval deviates from the interval 3:4 in the sense of approaching 471 cents. The black line added in the graph of course refers to diminution whereas the double drawn line added under argument (3) refers to desired augmentation. But when it is a question of merely counting "acceptation cases" among deviations presented in both directions, as in the statistical table to be explained, the double-drawn and the black line simply add to one another.

It is clear, then, that the four factors here mentioned may easily suffice to make statistically the acceptability of a given deviation highest for the sixth, next for the third, less for the fourth and least for the fifth. An appeal to habituation to tempered intervals as an explanatory factor is not called for by those experimental results, is far-fetched, and fifty years out of date because it ignores the accumulated tone psychology of today as represented by the four arguments above.

Let us say again what it is the purpose of this appendix to point out, that the biochemical affinities of tones can not plausibly be said to be products of habituation, and that the equal temperament of our musical instruments is no more than the reverberation in the ancestral caves an instance of such habituation becoming a determinant of our esthetic sound appreciation. The report that you can make any isolated musical interval, no matter how queer an interval you choose, appreciated by an individual provided only you let him listen to that particular isolated interval frequently enough in his life, is a myth. If it were more than a myth, if it were a biological possibility, the cuckoos would long ago have agreed on a definite interval. As a matter of fact the individuals of that species go on singing any kind of interval from a diminished minor third to an augmented fourth. There is something lacking in the chemistry of their nervous system which most individuals of the human species inherit. They possess the biological property of vocality, but not that of tonality. They are tone-deaf, like certain human beings who cannot carry a tone.

APPENDIX 14. THE FUTURE OF OUR MUSIC AND THE MUSIC OF THE FUTURE

The trend of music for centuries has been in the direction of greater and greater elaboration. Out of music thru its simplicity useful for accompanying a "country dance" developed the symphony merely to be listened to. Out of the originally quite small, little time consuming symphony developed the modern "tone picture" ("tone movie" would be better), "tone drama" or "tone panorama" lasting for an hour or two.[66] Of course, such an extended experience can not be in a psychological sense *unitary*. It can have unity only in a logical sense by rationalization. Such music is composed, that is, held together, only because certain *logical* systems are apparent in the *written* score.

That the numerous little units are held together in the score by something which is *extrinsic* to the psychology of music, is quite well understood by a painter who illustrated the fact quite recently. In the *Saturday Evening Post* of December 8, 1928, p. 77, there is "An Interpretation of Stravinsky's 'Le Sacre du Printemps'," painted for the Steinway Collection by Sergei Soudeikine. The painting seen there is composed of bits of visual perception which are held together only thru the force of rationalization, in this case, thru the force of artificial crossing lines which dominate the picture. No biological creature goes thru its struggle for life in that kind of space. Such crossing lines as displayed by Soudeikine are a species of geometrical rationalization which is extrinsic to concrete (artistic) visual perception, extrinsic to the reality of biology, intrinsic only to the *abstract* artificialities of scientific, i. e., engineering, design.

Such a design has the same relation to the ordinary functioning of the human eye as the relation which the fly-wheel of a giant power house would have to the muscle power of an individual human being who had been asked to turn that wheel over with his hand or his foot as if it were his great-grandmother's spinning wheel. Art can not well be abstract; sense experience must remain concrete in order to remain art.

If our music continues to develop along the line of greater and greater elaboration, it will become increasingly an esoteric game of a small coterie of musicians trained in abstractions. There is indeed no limit to the elaboration of abstractions, as is exemplified by the ease with which mathematicians transgress the concreteness of three-dimensional geometry. But that is not art.

Do not contradict the forecast of the beginning of the last paragraph by pointing to the crowds which fill the metropolitan concert rooms. How many of them in honesty have to record the price of the ticket in their accounts

66 Compare for analogy's sake the remarks made by Adolf Hildebrand on the lack of esthetic unity in the visual panorama, *The Problem of form in Painting and Sculpture*, translation by Meyer and Ogden, p. 56. When "technique" or technology becomes overpowering, true art suffers. True art is simple just as true sport is simple. The true sportsman-hunter does not take a machine gun along.

under the item "social prestige"? How many pretending to enjoy the music succeed in this pretense because their neighbors pretend no less? The naked in a society of the naked knows that he is safest when making the least fuss about the nakedness of the others who like him pretend to be richly clothed in musical understanding. So our music has a chance to continue on its road toward greater elaboration and toward becoming more and more an institution publicly supported and yet an esoteric game of a small coterie of musicians trained in abstractions,—just like out futuristic paintings, maybe.

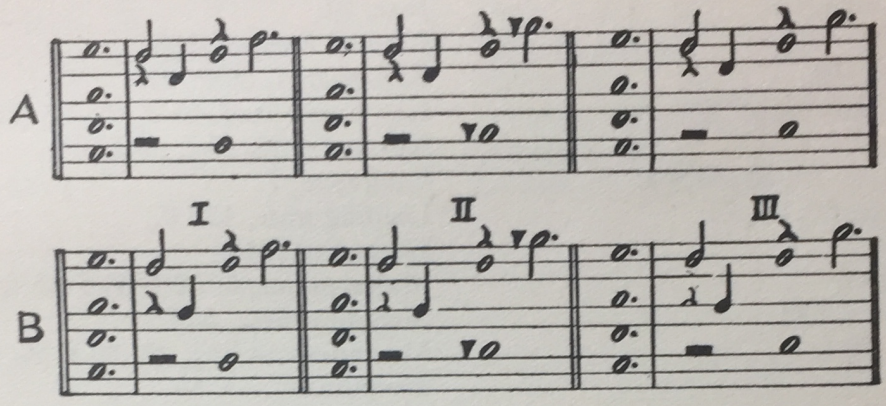
We have given an answer to the question "The Future of Our Music?" Before we give an answer to the question "The Music of the Future?" we ought to define Music with a capital M (Music for its own sake) as something in which at least nine-tenths of the population can really and want to participate. And yet let us agree not to include in this definition of Music mere stirring, mere sleep-preventing devices to the extent to which they are introduced by up-to-date contractors to increase the output of bricklayers; nor mere tempo-keeping devices serving mainly for the efficiency of marchers, dancers, pile drivers and other slaves. Of course, a dancer, e. g., *can* be regarded as a slave of his internal master (his sporting habits) as much as a pile driver can be regarded as a slave of an external master (his employer). Their music is not music for Music's sake.

Psychologists have not yet devised an exact test by which to measure how much enjoyment a person fished out of the pool of sensations to which he was a victim during the attendance at a "musicale," during which he was forced to sit still and shut up. I am therefore forced to make a guess.

I guess that nearly ten per cent of the population will never, no matter how skilful our public education becomes, be able honestly to enjoy even such "easy" music as a church hymn (removed from its religious application) or a folksong with the simplest accompaniment (not used for dancing) or an instrumental piece of similar simplicity (played altogether for its own sake). I said "enjoy." I mean "get their money's and time's worth out of mere listening." Only the remaining ninety per cent concern us further.

If by Music is meant the music *of* the fully ninety per cent made *by* the fully ninety per cent *purely for* music's sake, then we can dare pronounce something about the future of this Music. This Music in the future will never pass thru any revolution because for the vast majority among, if not indeed for all, these ninety per cent it has already exhausted the biological possibilities in "rules of the game" as well as in "extending the number of games for one sitting." It is like playing chess, only that, there, *new rules* might be agreed on. What corresponds to the rules of chess are the biologically fixed affinities. They result from the chemistry of our nervous system and no new ones can therefore "be agreed on" by habituation of some sort; and they have already all been brought into play in virtually all possible combinations of

twos and threes and fours and fives—just as all combinations of two or three moves in chess have probably been played. (Remember that we are not speaking of "written" abstractions.)



AN ADDED VARIATION MADE POSSIBLE BY A QUARTONE INSTRUMENT

A few, very few, *new* combinations of the particular kind which musicians call "variations" would probably be made possible by placing at the musician's disposal *a quartertone instrument.* In the two examples here given there are three variations in case A and three variations in case B. But the musician restricted to the ordinary piano can not use variation II in A or B and is restricted in either A or B to the variations I and III. The genius of a composer, if able to use a quartertone instrument, can give a slight melodic elaboration and an adequate harmonization to variation II as well as to I and III and thus increase his fund of possible variations of a theme. But to call *that* a "revolution" of Music would be an exaggeration not to be expected of a person of good judgment. ("Esoteric" music in written abstractions may be revolutionized, but does not concern us here.)

Summing up: The Music of the ninety per cent of the population left after eliminating those who are entirely "tone deaf," will remain in the future *essentially* what it is today and what it has been for some time in the past. What it may be millions of years hence, is a question about which we need not worry. But let the composer be discouraged as little as the chess player. It is still possible to play a game of chess, which, as a whole, may never have been played yet in the history of the world; and it is still possible to invent melodies which, regarded as wholes, may never have been invented yet in the history of the world.

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